MODELS OF ADVERTISEMENT AND CLIENTS ATTENDANCE OF THE MARKET USING A FOURIER–SERIES APPROXIMATION

TIRGUS REKLĀMAS UN KLIENTU APMEKLĒJUMA MODEĻI, IZMANTOJOT FURJĒ RINDU TUVINĀJUMUS

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Abstract. *The analysis of advertising impact on the competitiveness and profit of the companies is given. The model of the companies, which agreed to implement product supplies with the same fixed price and have constant agreed expenditures, is considered. As the principle of optimality the company's integral profit is studied. Fraction of the company's expenditures elasticity profit is given. The paper develops a simple model for economic time – varying series presenting the data concerning number of the market attendances. The method of modeling uses a Fourier approximation to present series of the market attendances. The Fourier approximation allows for implementing of the selection of frequencies to be included in the model. It is shown that approach for the modeling provides approximation of the economic time – varying series with a form of nonlinearity. Using daily data over the period of 2008:3– 2011:3, we have developed a model showing the nonlinear time – varying dependence of the variable presenting sequence of the number of market attendances.*

Keywords: *Profit, Advertising, Company, Competitiveness Model, Economic Time Series, Market Attendances, Fourier Approximation, Nonlinearity.*

1. Modeling of the clients attendance of the market using a Fourier – series approximation

1.1 Introduction

Following the approach of Ralf Becker, Walter Enders and Stan Hurn (2006), the economic time series model based on the Fourier approximation is studied. To implement this approximation we use trigonometric functions to approximate the unknown functional form. The approach based on the use of the Fourier approximation allows the substantiation as follows. It is known that Fourier approximation provides capturing the variation in any absolutely integrable function of time. In addition, time series presenting the number of attendances of the market is smooth and gradual. Apparently, Fourier approximation is particularly adept at modeling this type of time variation. The use of the Fourier approximation is now deeply studied in econometric research of Gallant (1984), Gallant and Souza (1991) and Ralf Becker, Walter Enders and Stan Hurn (2006). Current studies used one or two frequency components of a Fourier approximation to demonstrate the behavior of studying functional form. We showed that the simple model with one or two frequency allows to present Fourier approximation of the time varying intercept presenting the number of attendances of the market.

1.2 Modeling with a Fourier approximation

Let us denote time–varying variable presenting the number of the market attendances as an absolutely integrable α(t) function of time α_t approximated with Fourier series

$$\alpha_t = a_0 + \sum_{k=1}^{m} (a_k \sin 2\pi kt / T + b_k \cos 2\pi kt / T)$$

where m refers to the number of frequencies contained in the process generating a(t), k represents a particular frequency and T is the number of usable observations concerning the number of the market attendances. It is presented on the Fig.1.

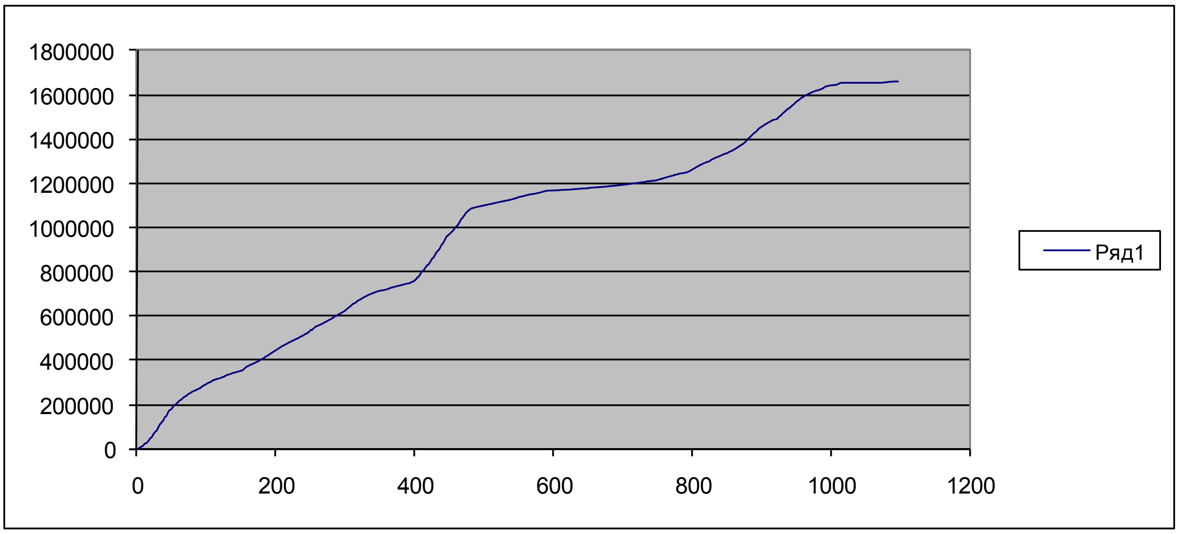

Figure 1. Curve presenting the number of the market attendances. (Vertical axis is the number of market attendances, horizontal axis is the day of the registering of market attendances).

Model 1(Fourier approximation). For the Model we showed that k=0,15 for sin and k=0,1 for cos. Consequently

2π0,15/ 1096=0,0008594 and 2π0,1/ 1096=0,0005729 thus $\alpha_t =$ 2287491,73sin0,001146t +1523626,405 cos0,001146t −1444188, where standard errors are as follows: 26164,1857for the coefficient of sin, 108590,3985for the coefficient of cos and 20480,67 for the constant;
t–statistics are as follows: 87,4283554 for the coefficient of sin, 14,03094957 for the coefficient of cos −12,83003 for the constant, R^2 =0,982132809; F=30040,28896.

The degree of freedom equals 1094. The curve presenting the Model is given in Fig.2.

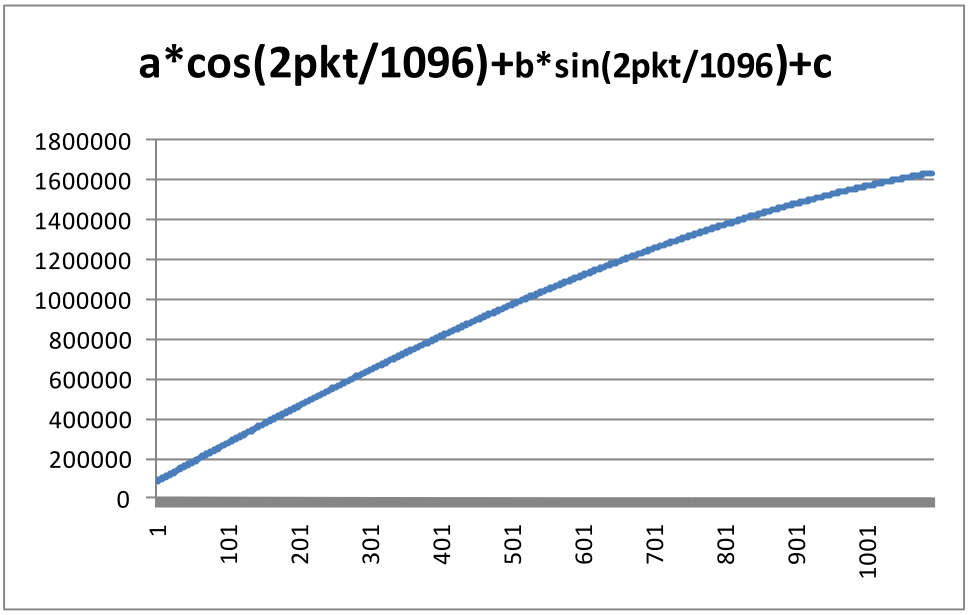

Figure 2. The curve presenting the Model 1. (Vertical axis is the number of market attendances; horizontal axis is the day of the registering of market attendances).

Model 2 (Quadratic trend). For the Model 2 we showed the quadratic trend of time series presenting the number of market attendances. That is $\alpha_t = -0{,}618657449\, t^2 + 2125{,}248521 t + 64367{,}76199$, where standard errors are as follows: 0,020330747 for the coefficient of t^2, 23,03294854 for the coefficient of *t*, 5470,8085 for the constant; t–statistics are as follows: 30,42964738 for the coefficient of t^2, 92,26992878 for the coefficient of *t*, 11,76567602 for the constant. $R^2 =$ 0,983246064; F=32072,7012. The degree of the freedom equals 1094. The curve presenting the Model 6 is given in Fig.4.

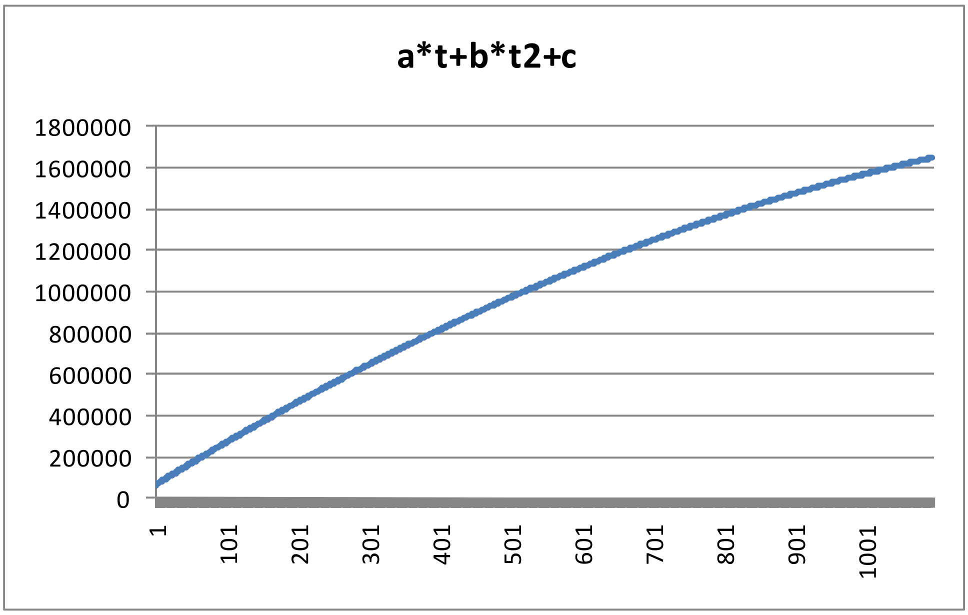

Figure 3. The curve presenting the Model 2.
(Vertical axis is the number of market attendances; horizontal axis is the day of the registering of market attendances).

Model 3. (Model with quadratic trend and Fourier time series). For the Model 3 we showed the existence of e time series approximation with quadratic trend and Fourier time series. It allows the presentation as follows.

α_t = 53076090,07 sin0,001146t +607711081,4 cos0,001146t – 43994,6013t–102,3537t^2 +–607642910, where k=0,15 for sin and k=0,1 for cos. Standard errors are as follows: 4446292,97 for the coefficient of sin, 42656152,94 for the coefficient of cos, 3910,37946 for the coefficient of t, 7,339236 for the coefficient t^2 and 42651630 for the constant; t–statistics are as follows: 11,93715539 for the coefficient of sin, 14,24673909 for the coefficient of cos, −11,2507243 for the coefficient of t, 13,94609 for the coefficient t^2 and −14,246651 for the constant.

R^2 = 0,987782619, F=22052,00239. The degree of freedom equals 1094. The curve presenting the Model 3 is given in Fig.4.

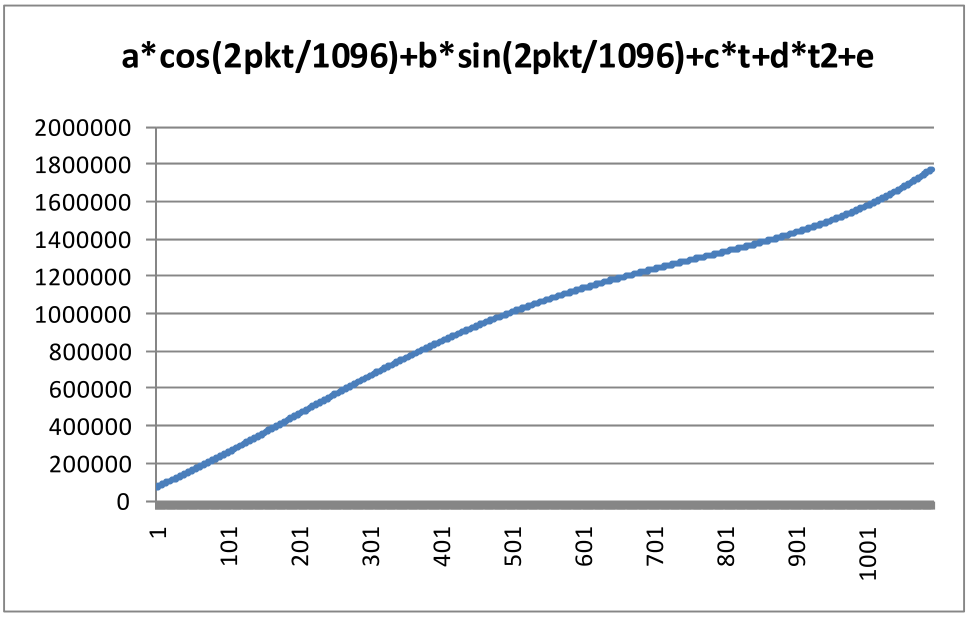

Figure 4. The curve presenting the Model 3.
(Vertical axis is the number of market attendances; horizontal axis is the day of the registering of market attendances).

2. Advertising and profit

2.1 Introduction

The set of factors affecting the customer's decision–making on expenses is variable and particularly advertising is one of the actions providing strong impact [4] on competitiveness of the company. Advertising modeling and analyzing show that it is a valuable mechanism allowing information distribution concerning the state of the market. The paper [4] is devoted to study of the approach, based on research of the market competition. This research is based on the study of companies' advertising activity. In [4] the principle of advertising modeling is studied and it is based on the consideration of the principle of individual optimality. The present paper studies the advertising modeling problem based on the global market principle of optimality. The paper complements the research done in [4] and continues the study of advertising efficiency problem investigation. The advertising modeling allowed to study and to calculate expense elasticity of the profit. It is characterized as advertising property and is given at the end of the paper.

2.2 Main definitions and concepts

Assume [1] that with fixed price P the set of companies $I = \{1,2,\ldots,N\}$ sells the product and all these companies have specific expenses equal to c. A_j and q_j are expenses for advertising and sales capacity of the j company correspondingly, $j \in I$. ρ_j and α_j are constants from the segment [0,1]. The higher is ρ_j the higher is mutual substitutability of advertising realized by the company j, $j \in I$. The coefficient α_j is given for assessment of the market demand as advertising expenses elasticity of the market demand.

2.3. Main model

If prices are equal and fixed, the demand is given through the following expression:

$$\sum_{i=1}^{N} q_i = K\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho}. \quad (1)$$

The profit is given as follows:

$$\sum_{i=1}^{N} \pi_i = (P-c)K\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho} - \sum_{i=1}^{N} \left(A_i\right) . \quad (2)$$

2.4. Analyzing the advertising management process

If constraints (2) are given then the Lagrangian function could be presented as follows:

$$L = \sum_{i=1}^{N} \pi_i + \lambda\left(\sum_{i=1}^{N} q_i - K\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho}\right). \quad (3)$$

Thus, from (2) and (3) follows that

$$L = (P-c)K\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho} - \sum_{i=1}^{N} \left(A_i\right) + \lambda\left(\sum_{i=1}^{N} q_i - K\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho}\right). \quad (4)$$

Consequently, partial derivatives of Lagrangian function (4) relatively to the expenses of the companies l and m, where $l, m \in I$, and equalizing this derivatives to 0 and as a result we have equation (5) .

$$\frac{\partial L}{\partial A_l} = (P-c)K\frac{\alpha}{\rho}\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho - 1} \rho\left(A_l\right)^{\rho-1} - 1 - \lambda K\frac{\alpha}{\rho}\rho\left(A_l\right)^{\rho-1}\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho - 1} = 0. \quad (5)$$

From this it follows, that

$$(P-c)K\alpha\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho - 1} \left(A_l\right)^{\rho-1} - 1 = \lambda K\alpha\left(A_l\right)^{\rho-1}\left(\sum_{j=1}^{N} \left(A_j\right)^{\rho}\right)^{\alpha/\rho - 1}. \quad (6)$$

Partial derivative of the Lagrangian (4) relatively to A_m after equalizing to 0 gives the following equation:

$$\frac{\partial L}{\partial A_m} = (P-c)K\frac{\alpha}{\rho}\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}\rho(A_m)^{\rho-1} - 1 - \lambda K\frac{\alpha}{\rho}\rho(A_m)^{\rho-1}\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1} = 0 \,. \quad (7)$$

Consequently,

$$(P-c)K\alpha\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}(A_m)^{\rho-1} - 1 = \lambda K\alpha(A_m)^{\rho-1}\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1} . \quad (8)$$

From (6) and (8) we have (9)

$$\left(\frac{A_l}{A_m}\right)^{\rho-1} = \frac{(P-c)K\alpha\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}(A_l)^{\rho-1} - 1}{(P-c)K\alpha\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}(A_m)^{\rho-1} - 1} . \quad (9)$$

Advertising expenses elasticity of the profit of the company l, $l \in I$ take the value

$$e_{\sum_{i=1}^{N}\pi_i, A_l} = \frac{\partial\sum_{i=1}^{N}\pi_i}{\partial A_l}\times\frac{A_l}{\sum_{i=1}^{N}\pi_i} = \frac{\left((P-c)K\frac{\alpha}{\rho}\rho\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1} - 1\right)A_l}{(P-c)K\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho} - \sum_{i=1}^{N}(A_i)} . \quad (10)$$

Advertising expenses elasticity of the profit of the company m, $m \in I$ take the value

$$e_{\sum_{i=1}^{N}\pi_i, A_m} = \frac{\partial\sum_{i=1}^{N}\pi_i}{\partial A_m}\times\frac{A_m}{\sum_{i=1}^{N}\pi_i} = \frac{\left((P-c)K\frac{\alpha}{\rho}\rho\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1} - 1\right)A_m}{(P-c)K\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho} - \sum_{i=1}^{N}(A_i)} . \quad (11)$$

From (10) and (11) we have

$$\frac{e_{\sum_{i=1}^{N}\pi_i, A_l}}{e_{\sum_{i=1}^{N}\pi_i, A_m}} = \frac{(P-c)K\alpha\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}(A_l)^{\rho-1} - 1}{(P-c)K\alpha\left(\sum_{j=1}^{N}(A_j)^{\rho}\right)^{\alpha/\rho-1}(A_m)^{\rho-1} - 1}\times\frac{A_l}{A_m} . \quad (12)$$

From (9) and (12) we have

$$\frac{e_{\sum_{i=1}^{N}\pi_i, A_l}}{e_{\sum_{i=1}^{N}\pi_i, A_m}} = \left(\frac{A_l}{A_m}\right)^{\rho} . \quad (13)$$

Thus we showed that the companies' advertising expenses elasticity of profits is proportional to the advertising expenses.

3. Data

As data we used time series presenting the number of market attendances recorded during two years period for the company in a competitive environment. Considered statistics consist of the following data: daily capacity of recorded number of market attendances as output of advertising of heating equipment. The dynamics is given in Fig. 1. Trends and Fourier approximations are given in the Model 1–Model 3 as well in Fig. 2–Fig.4. Thus, the dynamics of studied factors showed that the curves are oscillated and stable. We showed that the Model 3 presents approximation with the type fitted to the source curve presented in Fig. 1.

4. Conclusions

In our research, we have proposed advertising modeling mechanism allowing assessment of the expenses elasticity of profits as well as providing the calculation of advertising expenses, which are optimal to get the highest level of profit. By introducing the principle of optimality as global advertising principle of optimality we set up the problem of optimality.

The study of data of the Company time series presenting the number of market attendances recorded during two years period for the company in competitive environment allowed for developing of the model as approximation using Fourier time series. Thus, one of the goals of the present study devoted to modeling of the series presenting the number of market attendances is to show that use of Fourier approximation allows studying the behavior of time series presenting the number of market attendances.

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Kopsavilkums

Tiek pētīts ekonomikas laikrindu modelis, kura pamatā ir Furjē tuvinājumi. Nezināmas funkcionālas formas funkciju tuvinājuma īstenošanas pieejas pamatā ir trigonometrisko funkciju izmantošana. Šī pieeja, kuras pamatā ir Furjē tuvinājumu izmantošana, pieļauj šādu pamatojumu. Furjē tuvinājumi nodrošina jebkuru izmaiņu uztveršanu jebkurā absolūti integrējamā laika funkcijā. Kā parādīja laikrindas, tad

tirgus apmeklējumu skaits ir vienmērīgs un pakāpenisks. Pēc ekonomikas laikrindu izpētes, kas īstenota dažādos pētījumos, mēs izmantojām Furjē tuvinājumu vienu vai divus frekvences komponentus, lai parādītu izturēšanos funkcionālās formas izpētē. Mēs parādījām, ka vienkāršs modelis ar vienu vai diviem frekvencēm ļauj parādīt Furjē tuvinājumu laika ziņā mainīgai aizturei, kas parāda tirgus apmeklējumu skaitu.

Klienta lēmuma pieņemšanu par izdevumiem ietekmē dažādu faktoru kopums, un jo īpaši reklāma ir viens no pasākumiem, kas stipri ietekmē uzņēmuma konkurētspēju. Zināms, ka reklāmas modelēšana un analīze ir vērtīgs mehānisms, kas ļauj izplatīt informāciju par situāciju tirgū. Šī pētījuma pamatā uzņēmumu reklāmas darbības izpēte. Šajā darbā tiek pētīta reklāmas modelēšanas problēma, pamatojoties uz pasaules tirgus optimalitātes principu. Šis darbs papildina pētījumu, kas veikts, lai izpētītu atsevišķu optimalitāti un turpina pētīt reklāmas efektivitātes problēmu izmeklēšanu. Reklāmas modelēšana ļāva izpētīt un aprēķināt peļņas izdevumus elastību.

Kā dati izmantotas laikrindas, kuras parāda tirgus apmeklējumu skaitu, kas reģistrēts divu gadu laikā, kopš uzņēmums darbojas konkurences apstākļos. Aplūkoto statistiku veido šādi dati: tirgus apmeklējumu reģistrētā skaita apjoms dienā kā iznākums no siltumiekārtu reklāmas. Izpētīto faktoru dinamika parādīja, ka līknes ir svārstīgas un stabila. Mēs parādījām, ka izstrādātais modelis parāda tuvinājumu ar to veidu, kas pielāgots avota līknei.

Mūsu pētījumā mēs ierosinājām reklāmas veidošanas mehānismu, kas ļauj novērtēt peļņas izdevumu elastību, kā arī nodrošināt reklāmas izdevumu aprēķinu, kas ir optimāli, lai iegūtu vislielāko peļņu. Ieviešot optimalitātes principu par vispasaules optimalitātes reklāmas principu, mēs nosakām optimalitātes problēmu.

Uzņēmuma laikrindu datu pētījumā tiek parādīts tirgus apmeklējumu skaits divu gadu laikā, kopš uzņēmums darbojas konkurences apstākļos, kas ļāva izstrādāt modeli kā tuvinājumu, izmantojot Furjē laikrindas. Tādējādi viens no šī pētījuma mērķiem, kas veltīts šo rindu modelēšanai un parāda tirgus apmeklējumu skaitu, ir demonstrēt, ka Furjē tuvinājumu izmantošana ļauj izpētīt laikrindu izturēšanos, parādot tirgus apmeklējumu skaitu.