

# *A unified model for analyzing dimensional relationships*

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**Abstract.** The paper presents an attempt to introduce an integrated model for analysing dimensional relationships. The basis for the application of a unified approach in revealing, analysing and solving the dimensional chains is the fact that each element of the dimensional relationships in the product has a spatial expression. The practical side of the proposed unified model for the analysis of dimensional relationships is also presented. The essence of this model is that a single approach is applied to solve all dimensional chains, regardless of their spatial location. The model represents the dimensional chain by the coordinates of a sequence of basic points, the distances between which determine the components of the dimensional chain.

**Keywords:** Mechanical engineering technology, dimensional chains, dimensional analysis, CAD-CAM systems.

## I. INTRODUCTION

In mechanical engineering practice, the sizing of products is carried out in the three coordinate planes with dimensions, parallel to the coordinate axes. Thus, the dimensional relationships in each of the coordinate planes are represented by dimensional chains, composed of parallel components (dimensions) [1]. Decisive role for this approach in construction has the technical and technological level of production techniques. The guiding principle is to have details with a simple geometric shape, which does not require the use of production machines with complex construction and kinematics. Having in mind that such machines, built with hard kinematics, are very expensive, the application of the above-mentioned principle is easy to explain for the reason of achieving an acceptable cost price of the products. It should be definitely said that the modern level of production technology has significantly outstripped the logic of the construction thinking, and the limitation of the simple geometric shape has no significant practical value. Modern machines have flexible kinematics, conditioned by independent activation of their working elements, the synchronization between which is realized by means of computer logic

and computer control [2]. For this reason, there are no technical limitations for production of details with a complex shape and relative position of their surfaces, as well as of a complex spatial relationship between the individual details in the product.

The simple geometric shape principle in construction also reflects in the scientific approach to studying the dimensional relationships in the products [3]. Solutions have been sought, in which the problem is reduced to a uniaxial model of the dimensional relationships – dimensional chains with parallel components. This type of a model is theoretically well developed. In cases where the dimensional chain contains components with an angular position with respect to the remaining components, transformation of the dimensional chain into a uniaxial one is applied, by projecting it onto the coordinate axes. Such examples are shown in Fig.1 and Fig.2.

In the first case, a projection on an axis, parallel to the closing component is considered. The advantage is the simplicity of the model. The disadvantage is that it does not reflect the influence of the components, located perpendicular to the closing component. This problem has been overcome in the second model, where two uniaxial chains are obtained, which are projections on the two coordinate axes. However, problems arise here also in determining the tolerances of the constituent components.

Even more complicated is the case with 3D (three-dimensional) spatial dimensional chains [4]. It is considered that they can be studied as three uniaxial chains, which are projections of the spatial chain onto the three coordinate axes [5], [6].

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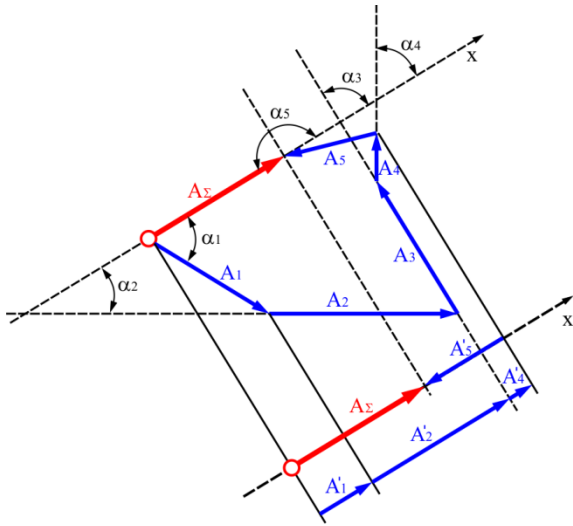


Fig. 1. Transformation of the dimensional chain into a uniaxial one

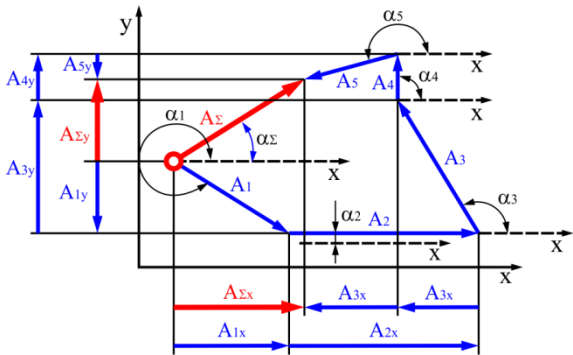


Fig. 2. Transformation of the dimensional chain into two uniaxial chains

It should be noted that regardless of these ideas for solving dimensional chains with non-parallel components, the issue has not yet been studied sufficiently well and no applied algorithms have been developed for use in product design.

## II. MATERIALS AND METHODS

The basis for the application of a unified approach in revealing, analyzing and solving the dimensional chains is the fact that each element of the dimensional relationships in the product has its spatial expression. A dimensional chain is a closed loop of serially connected dimensions. Each of them is a line segment (part of a straight line), whose size and position in space are determined by the coordinates of its endpoints.

The line segment  $A_i$ , with a general position in space, is shown in Fig. 3. It is defined by the points  $M_i(x_i, y_i, z_i)$  and  $M_{(i-1)}[x_{(i-1)}, y_{(i-1)}, z_{(i-1)}]$ . Its length is obtained from the sum:

$$A_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \quad (1)$$

The differences in this sum are its projections on the coordinate axes:

$$\begin{aligned} x_i - x_{i-1} &= A_{ix}; \\ y_i - y_{i-1} &= A_{iy}; \\ z_i - z_{i-1} &= A_{iz}. \end{aligned} \quad (2)$$

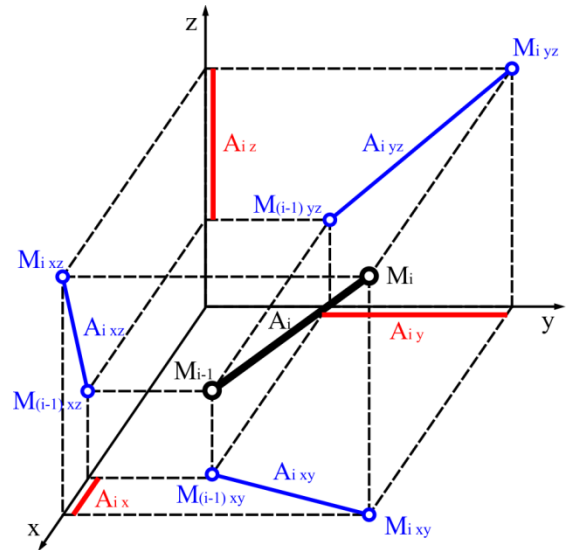


Fig. 3. A line segment in space

From this example it can be seen that if one of the coordinates of the constituent components in a dimensional chain is a constant, then the chain will be located in a plane, perpendicular to the corresponding coordinate axis. For example, with  $z_i = \text{const}$ , it will lie in a plane, parallel to  $xOy$ . If two of the coordinates are constant, the dimensional chain will have parallel components, i.e., it is a uniaxial dimensional chain. From here it follows that the dimensional chains with parallel components and the planar dimensional chains are particular cases of the spatial dimensional chains. Therefore, they obey a common theoretical model.

### A. Revealing the dimensional relationship

The task of compiling and solving a dimensional chain will be presented in a general form. Fig. 4 shows a spatial dimensional chain with non-parallel dimensions.

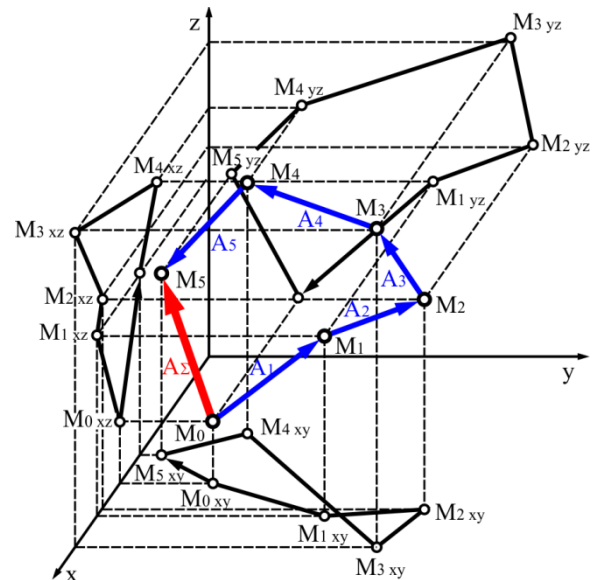


Fig. 4. Spatial dimensional chain

The closing component is the dimension  $A_\Sigma$ , and the remaining dimensions are constituent components. The chain is constructed using the vector sum principle. The constituent components make up the vector polygon, and

the closing component is the resultant vector. In this case:

$$\vec{A}_\Sigma = \sum_{i=1}^n \vec{A}_i. \quad (3)$$

When projecting the dimensional chain on the three coordinate planes, three planar dimensional chains with non-parallel components are obtained.

The dimensional chain is identified by the coordinates of the points  $M_0; M_1; M_2; \dots; M_n$ .

The distances between them are the constituent components  $A_1; A_2; A_n$ .

The closing component is defined by the points  $M_0; M_n$ .

Therefore:

$$\begin{aligned} A_{\Sigma x} &= x_n - x_0; \\ A_{\Sigma y} &= y_n - y_0; \\ A_{\Sigma z} &= z_n - z_0. \end{aligned} \quad (4)$$

For the size of the closing component, it is obtained:

$$A_\Sigma = \sqrt{A_{\Sigma x}^2 + A_{\Sigma y}^2 + A_{\Sigma z}^2}. \quad (5)$$

The angular position of the closing component is determined by the angles, relative to the three coordinate axes:

$$\begin{aligned} \alpha_{\Sigma x} &= \arccos \frac{A_{\Sigma x}}{A_\Sigma}; \\ \alpha_{\Sigma y} &= \arccos \frac{A_{\Sigma y}}{A_\Sigma}; \\ \alpha_{\Sigma z} &= \arccos \frac{A_{\Sigma z}}{A_\Sigma}. \end{aligned} \quad (6)$$

### **B. Dimensional tolerances in the dimensional chain**

#### a) Limited summation – “Max - Min” method

Any change to a component in the dimensional chain also changes the closing component. If the dimension of the component  $A_i$  changes by  $\Delta A_i$ , a change will occur in the coordinates of the point  $M_i$ :

$$\Delta x_i = \Delta A_i \cdot \xi_{ix}; \Delta y_i = \Delta A_i \cdot \xi_{iy}; \Delta z_i = \Delta A_i \cdot \xi_{iz}, \quad (7)$$

where:

$$\xi_{ix} = \frac{A_{ix}}{A_i}; \xi_{iy} = \frac{A_{iy}}{A_i}; \xi_{iz} = \frac{A_{iz}}{A_i}. \quad (8)$$

The same change will occur in all the other basic points, and respectively, in the endpoint of the chain.

Given that each component of the chain will have a different size for the specific product, the coordinates of the point  $M_n$ , and respectively the change of the closing component, will be:

$$\Delta_{\Sigma x} = \sum_{i=1}^n \Delta A_i \xi_{ix};$$

$$\Delta_{\Sigma y} = \sum_{i=1}^n \Delta A_i \xi_{iy}; \quad (9)$$

$$\Delta_{\Sigma z} = \sum_{i=1}^n \Delta A_i \xi_{iz}.$$

The resizing of the closing component will be:

$$\Delta_\Sigma = \sqrt{\Delta_{\Sigma x}^2 + \Delta_{\Sigma y}^2 + \Delta_{\Sigma z}^2}. \quad (10)$$

The closing unit size variance will be within the limits:

$$\omega_\Sigma = \Delta_{\Sigma \max} - \Delta_{\Sigma \min}. \quad (11)$$

Given that the number (10) is positive ( $\Delta_\Sigma \geq 0$ ) and the maximum change of each of the components is within the dimension tolerance ( $\Delta_{i \max} = T_i$ ), from the expressions (9) and (10) it is obtained:

$$T_\Sigma = \sqrt{\left(\sum_{i=1}^n T_i |\xi_{ix}|\right)^2 + \left(\sum_{i=1}^n T_i |\xi_{iy}|\right)^2 + \left(\sum_{i=1}^n T_i |\xi_{iz}|\right)^2}. \quad (12)$$

In the design task, the tolerance of the closing component is distributed among the constituent components according to the principle of the same accuracy class, in which the same number  $\beta_{cp}$  of the tolerance units is defined for all constituent components. The tolerances of the constituent components are determined by the calculated average number of tolerance units:

$$T_i = \beta_{cp} \cdot E_i. \quad (13)$$

From the expressions (12) and (13),  $\beta_{cp}$  is found:

$$\beta_{cp} = \frac{T_\Sigma}{\sqrt{\left(\sum_{i=1}^n E_i |\xi_{ix}|\right)^2 + \left(\sum_{i=1}^n E_i |\xi_{iy}|\right)^2 + \left(\sum_{i=1}^n E_i |\xi_{iz}|\right)^2}}, \quad (14)$$

where  $E_i$  is the size of the tolerance unit. It is standardized and determined by the expression:

$$E_i = 0,45(A_{\min} \cdot A_{\max})^{\frac{1}{6}} + 0,001(A_{\min} \cdot A_{\max})^{\frac{1}{2}} \quad (15)$$

where  $A_{\min}, A_{\max}$  are the limits of the size interval, in which  $A_i$  falls.

The calculated values of the tolerances are rounded, after which a check is performed to see if the inequality is fulfilled:

$$T_\Sigma \geq \sqrt{\left(\sum_{i=1}^n T_i |\xi_{ix}|\right)^2 + \left(\sum_{i=1}^n T_i |\xi_{iy}|\right)^2 + \left(\sum_{i=1}^n T_i |\xi_{iz}|\right)^2}. \quad (16)$$

In case of inequality failure, the roundings in the tolerances are corrected.

#### b) Probabilistic summation

In the theory of dimensional analysis [5] - [8], based on the theory of probabilities, it is assumed that, for  $n \geq 4$  the probability for all constituent components to combine with their extreme values is a statistically impossible event. For this case, the expression for the tolerance of the closing component has the form:

$$T_{\Sigma} = t_{\Sigma} \sqrt{\sum_{i=1}^n T_i^2 \xi_i^2 \lambda_i'} \quad (17)$$

where  $\lambda_i' = 1/t_i^2$  is the relative dispersion, which depends on the distribution law of the random variable  $A_i$ ;  $\xi_i$  - the transmission coefficient of the constituent component relative to the closing component.

The coefficient  $\lambda_i'$  is also defined as the ratio of the root-mean-square deviation and the scattering field, as it follows:

$$\lambda_i' = \left( \frac{2\sigma_i}{\omega_i} \right)^2$$

For the normal law  $\omega = 6\sigma$ , where  $\lambda_i' = 1/9$ . For the law of equal probability  $\sigma = (b-a)/2\sqrt{3}$  and  $\omega = (b-a)$ , whereupon  $\lambda_i' = 1/3$ . For the law of the triangle  $\sigma = a/\sqrt{6}$  and  $\omega = 2a$ , whereat  $\lambda_i' = 1/6$ .

Under the assumption that all constituent components are distributed according to the normal law ( $\lambda_i' = 1/9$ ) and a risk of waste of 0,27% ( $t_{\Sigma} = 3$ ) is accepted, equation (17) takes the form:

$$T_{\Sigma} = \sqrt{\sum_{i=1}^n T_i^2 \xi_i^2} \quad (18)$$

In 3D dimensional chains, the tolerance of the closing component will be:

$$T_{\Sigma} = \sqrt{\sum_{i=1}^n T_i^2 \xi_{ix}^2 + \sum_{i=1}^n T_i^2 \xi_{iy}^2 + \sum_{i=1}^n T_i^2 \xi_{iz}^2} \quad (19)$$

$$T_{\Sigma} = \sqrt{\sum_{i=1}^n T_i^2 (\xi_{ix}^2 + \xi_{iy}^2 + \xi_{iz}^2)}$$

Considering the fact, that:  $\xi_{ix}^2 + \xi_{iy}^2 + \xi_{iz}^2 = 1$ , from the expression (19) we obtain:

$$T_{\Sigma} = \sqrt{\sum_{i=1}^n T_i^2} \quad (20)$$

In the design task in expression (20) the tolerances of the constituent components are replaced by the expression (13), which gives:

$$T_{\Sigma} = \beta_{cp} \sqrt{\sum_{i=1}^n E_i^2} \quad (21)$$

The average number of the tolerance units in the case of probabilistic summation of the tolerances is:

$$\beta_{cp} = \frac{T_{\Sigma}}{\sqrt{\sum_{i=1}^n E_i^2}} \quad (22)$$

The tolerances of the constituent components are determined by the calculated average number of tolerance units:

$$T_i = \beta_{cp} \cdot E_i$$

The calculated values of the tolerances are rounded, after which a check is performed to see if the inequality is fulfilled:

$$T_{\Sigma} \geq \sqrt{\sum_{i=1}^n T_i^2} \quad (23)$$

In case of inequality failure, the roundings in the tolerances are corrected.

### C. Determining the average values of the tolerance zones

The average value of the tolerance zone defines the clustering center for the scatter of the corresponding dimension. The coordinate of this center is located on the dimension line and determines the deviation from the nominal size. It is accepted that the displacement of the average value of the tolerance zone with respect to the nominal size is denoted by EM. The relationship between the average values of the tolerance zones of the constituent components and of the closing component is analogous to the dependence (5):

$$EM_{\Sigma} = \sqrt{EM_{\Sigma x}^2 + EM_{\Sigma y}^2 + EM_{\Sigma z}^2} \quad (24)$$

where

$$EM_{\Sigma x} = \xi_{\Sigma x} EM_{\Sigma};$$

$$EM_{\Sigma y} = \xi_{\Sigma y} EM_{\Sigma}; \quad (25)$$

$$EM_{\Sigma z} = \xi_{\Sigma z} EM_{\Sigma}.$$

When solving the design task, the average value of the tolerance zone of the closing component is set, and those, of the constituent components must be determined. For this purpose, the average value for one of the components remains unknown, while for the others, the average values are chosen for constructive and (or) technological reasons. The unknown average value is determined by the equations (26):

$$EM_{\Sigma x} = \sum_{i=1}^n EM_{ix} = \sum_{i=1}^n EM_i \xi_{ix};$$

$$EM_{\Sigma y} = \sum_{i=1}^n EM_{iy} = \sum_{i=1}^n EM_i \xi_{iy}; \quad (26)$$

$$EM_{\Sigma z} = \sum_{i=1}^n EM_{iz} = \sum_{i=1}^n EM_i \xi_{iz}.$$

## III. RESULTS AND DISCUSION

### A. Example 1

Fig.5 shows the construction of a two-piece body with a gear. It is necessary to ensure during assembly the accuracy of the centre-to-centre distance, which is a closing component of the dimensional chain.

The initial construction data are:

$$A_{\Sigma} = 160 \pm 0,1 \text{ mm}; A_1 = 150 \text{ mm};$$

$$A_2 = 38,6 \text{ mm}; A_3 = 100 \text{ mm}; A_4 = 70 \text{ mm}.$$

To determine the coordinates of the basic points, the coordinate system must be defined. It is correct that the coordinate system coincides with the mounting base. It will also be used as a technological base for orienting the

workpiece during detail processing. Fig.5 shows the coordinate system and the coordinates of the basic points.

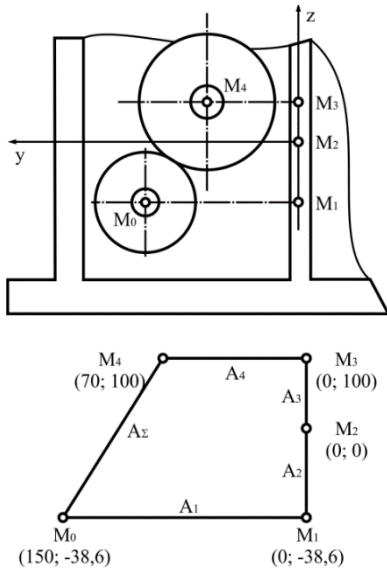


Fig. 5. Two-piece body with a gear.

a) Check for the nominal value of the closing component:

$$A_{\Sigma y} = y_4 - y_0;$$

$$A_{\Sigma y} = 70 - 150 = -80 \text{ mm};$$

$$A_{\Sigma z} = z_4 - z_0;$$

$$A_{\Sigma z} = 100 - (-38,6) = -138,6 \text{ mm};$$

$$A_{\Sigma} = \sqrt{A_{\Sigma y}^2 + A_{\Sigma z}^2};$$

$$A_{\Sigma} = \sqrt{(-80)^2 + 138,6^2} = 160,03 \text{ mm}.$$

b) Determining the tolerances of the constituent components

• *Magnitudes of the tolerance units:*

$A_1=150 \text{ mm}$	dimensional interval 120 to 180 mm	$E_1=2,52$
$A_2=38,6 \text{ mm}$	dimensional interval 30 to 50 mm	$E_2=1,56$
$A_3=100 \text{ mm}$	dimensional interval 80 to 120 mm	$E_3=2,17$
$A_4=70 \text{ mm}$	dimensional interval 50 to 80 mm	$E_4=1,86$

• *Transmission coefficients:*

$$\xi_{1y} = \frac{A_{1y}}{A_1} = \frac{y_1 - y_0}{A_1} = \frac{0 - 150}{150} = -1;$$

$$\xi_{1z} = \frac{A_{1z}}{A_1} = \frac{z_1 - z_0}{A_1} = \frac{-38,6 - (-38,6)}{150} = 0.$$

They are determined in a similar way for the other components:

$$\xi_{2y} = 0; \xi_{3y} = 0; \xi_{4y} = 1; \xi_{2z} = 1; \xi_{3z} = 1; \xi_{4z} = 0.$$

• *Average number of tolerance units in case of limited summation of the tolerances:*

$$\beta_{cp} = \frac{200}{\sqrt{(E_1 + E_4)^2 + (E_2 + E_3)^2}} = 34,76.$$

• *Dimensional tolerances:*

$$T_1 = \beta_{cp} \cdot E_1 = 34,76 \cdot 2,52 = 87,6;$$

$$T_2 = \beta_{cp} \cdot E_2 = 34,76 \cdot 1,56 = 54,23;$$

$$T_3 = \beta_{cp} \cdot E_3 = 34,76 \cdot 2,17 = 75,434;$$

$$T_4 = \beta_{cp} \cdot E_4 = 34,76 \cdot 1,86 = 64,66.$$

Rounded:

$$T_1 = 88 \text{ } \mu\text{m}; T_2 = 54 \text{ } \mu\text{m}; T_3 = 75 \text{ } \mu\text{m}; T_4 = 64 \text{ } \mu\text{m}.$$

Check:

$$T_{\Sigma} \geq \sqrt{(T_1 + T_4)^2 + (T_2 + T_3)^2} = 199,36.$$

• *Average number of tolerance units in probabilistic summation of the tolerances:*

$$\beta_{cp} = \frac{200}{\sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2}} = 48,54.$$

• *Dimensional tolerances:*

$$T_1 = \beta_{cp} \cdot E_1 = 48,54 \cdot 2,52 = 122,32;$$

$$T_2 = \beta_{cp} \cdot E_2 = 48,54 \cdot 1,56 = 75,72;$$

$$T_3 = \beta_{cp} \cdot E_3 = 48,54 \cdot 2,17 = 105,33;$$

$$T_4 = \beta_{cp} \cdot E_4 = 48,54 \cdot 1,86 = 90,28.$$

Rounded:

$$T_1 = 120 \text{ } \mu\text{m}; T_2 = 76 \text{ } \mu\text{m}; T_3 = 105 \text{ } \mu\text{m}; T_4 = 90 \text{ } \mu\text{m}.$$

Check:

$$T_{\Sigma} \geq \sqrt{120^2 + 76^2 + 105^2 + 90^2} = 201,04.$$

Correction:

$$T_1 = 120 \text{ } \mu\text{m}; T_2 = 80 \text{ } \mu\text{m}; T_3 = 100 \text{ } \mu\text{m}; T_4 = 90 \text{ } \mu\text{m}.$$

Check:

$$T_{\Sigma} \geq \sqrt{120^2 + 80^2 + 100^2 + 90^2} = 197,23.$$

c) *Average values of the tolerance zones*

It is accepted that:  $EM_2 = EM_3 = EM_4 = 0$ .

In this case from equations (26) we get:

$$EM_{\Sigma y} = \sum_{i=1}^n EM_{iy} = EM_{1y}; \quad EM_{\Sigma z} = \sum_{i=1}^n EM_{iz} = EM_{1z}.$$

Given as an initial condition  $EM_{\Sigma} = 0$ , it follows that  $EM_1 = 0$  as well.

d) Representation of dimensions by means of limit deviations

• *Limited summation*

$$A_1 = 150 \pm 0,044; A_2 = 38,6 \pm 0,027;$$

$$A_3 = 100 \pm 0,037; A_4 = 70 \pm 0,032.$$

• *Probabilistic summation*

$$A_1 = 150 \pm 0,06; A_2 = 38,6 \pm 0,04;$$

$$A_3 = 100 \pm 0,05; A_4 = 70 \pm 0,045.$$

### B. Example 2

The developed theoretical model will be applied to solving the dimensional chain, presented in Fig.4. The design task is solved, having the closing component set

$A_2=70\pm 0,1$  mm. The coordinates of all basic points are also set, with the exception of the point  $M_4$ . Thus, left for specifying is the dimension  $A_5$ , by means of which

achieving the set dimension of the closing component will be ensured. For clarity, the solution to the problem is presented in Table 1.

TABLE 1 SPATIAL DIMENSIONAL CHAIN

		X	Y	Z	$A_x$	$A_y$	$A_z$	A	xx	xy	xz	
	$M_0$	70	55	5								
$A_1$	$M_1$	110	95	50	35	40	30	61,03	0,57	0,66	0,49	
$A_2$	$M_2$	85	115	70	-25	20	25	40,62	-0,62	0,49	0,62	
$A_3$	$M_3$	125	100	100	40	-15	40	58,52	0,68	-0,26	0,68	
$A_4$	$M_4$	35	45	80	-85	-55	-20	103,20	-0,82	-0,53	-0,19	
$A_5$	$M_5$	50	25	65	15	-20	-15	29,15	0,51	-0,69	-0,51	
$A_5$					-20	-30	60	70,00	-0,29	-0,43	0,86	
	$A_{min}$	$A_{max}$	$A_{cp}$	$E_i$	$E_i^{*xx}$	$E_i^{*xy}$	$E_i^{*xz}$	$E_i^2$	$T_1$	$T_2$	$T_1$	$T_2$
									calculated		rounded	
$A_1$	50	80	63,25	1,86	1,06	1,22	0,91	3,45	43,94	93,52	45	90
$A_2$	30	50	38,73	1,56	0,96	0,77	0,96	2,44	36,96	78,66	35	80
$A_3$	50	80	63,25	1,86	1,27	0,48	1,27	3,45	43,94	93,52	45	90
$A_4$	80	120	97,98	2,17	1,79	1,16	0,42	4,72	51,43	109,46	50	110
$A_5$	18	30	23,24	1,31	0,67	0,90	0,67	1,71	30,95	65,87	30	65
					5,76	4,52	4,24	15,76				
					33,13	20,39	17,94	3,97				
						8,45						
			$T_s = 200$		$b_1 = 23,68$		$b_2 = 50,38$					

The algorithm for solving the task is as it follows:

1. The projections of the component  $A_5$  along the three coordinate axes are determined, and, respectively, the coordinates of the point  $M_4$ :

$$A_{5x} = -A_{\Sigma x} + \sum_{i=1}^4 A_{ix} = 15;$$

$$X_{M4} = X_{M5} - A_{5x} = 35;$$

$$A_{5y} = -A_{\Sigma y} + \sum_{i=1}^4 A_{iy} = -20;$$

$$Y_{M4} = Y_{M5} - A_{5y} = 45;$$

$$A_{5z} = -A_{\Sigma z} + \sum_{i=1}^4 A_{iz} = -15;$$

$$Z_{M4} = Z_{M5} - A_{5z} = 80.$$

2. The nominal dimensions of the constituent components are determined:

$$A_i = \sqrt{A_{ix}^2 + A_{iy}^2 + A_{iz}^2}.$$

3. The transmission coefficients are found by formula (8).

4. The magnitudes of the tolerance units are determined, using formula (15).

5. The average number of tolerance units  $\beta_1$  is determined by the max-min method - formula (14) and  $\beta_2$  is found by the probabilistic method - formula (22).

6. The tolerances  $T_1$  (for the max-min method) and  $T_2$  (for the probabilistic method) are calculated according to formula (13), after which they are rounded.

7. The average values of the tolerance zones are defined. In this case, the tolerance of the closing component is symmetrically located with respect to the nominal dimension, which is convenient to accept also for the constituent components, i.e.,  $A_i = A_i \pm 1/2T_i$ .

#### IV. CONCLUSIONS

The presented mathematical model and methodology for analyzing and solving the tasks of dimensional analysis, confirm the raised thesis of a unified approach to considering the dimensional chains with parallel components, planar and spatial dimensional chains. It is irrefutably proven that dimensional chains with parallel components and planar dimensional chains are special cases of the spatial dimensional chains. There is no need for a different approach when solving them, as they are the different sides of the same whole.

Defining a dimensional chain by means of the coordinates of the basic points presents a new approach to revealing and solving it. The basic points are the points of contact between the surfaces of the connected elements in the structure. They are known in result of the automated product design. They are also used for production of parts by means of CNC machine tools. Presented this way, dimensional analysis becomes a compatible and natural element of the modern CAD-CAM systems.

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