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Vector-Matrix Computer Method for Drafting Circling-Point Curves and Centering-Point Curves of Burmester

Marin Jordanov Marinov
Technical University - Gabrovo
Department of Industrial Design and Textile Engineering
Gabrovo, Bulgaria
marinov@tugab.bg

Abstract. **The author suggests a method for drafting circling-point curves and centering-point curves of Burmester in the present paper. The geometrical compulsion makes the best use of synthesis of levers mechanisms by vector-matrix computer method under existing circumstances.**
Propose the drafting of the Burmester circling-point and centering-point curves are more applicable and more approach for engineers and constructors in the manufacture.
Keywords: **circling-point curves of Burmester, centering-point curves of Burmester, vector-matrix computer method for synthesis linkage of bars.**

I. Introduction

The foundations of kinematic geometry, laid down by Burmester [1888], for the finitely distant discrete positions in general plane motion of a rigid body, are built from the position of the projection geometry. He created a theory according to which points on a moving plane have four discrete positions on a circle fixed in a stationary plane. Burmester's name is associated with the concepts known in the literature: circle point curves (CPC), center point curves (CPC), Burmester points and centers.

Graphical solutions for determining CPC and CCT proposed by Burmester [1888] and Alt [1921] lead to laborious graphical procedures and do not provide accuracy in reproducing these curves. However, the indicated difficulties are one of the reasons for the evolution of the task of determining and drawing the CPC and CCT, especially after the computer technology entered massively with its powerful computing and graphic capabilities.

The tasks for the synthesis of the mechanisms according to the method of Burmester [1888], were significantly expanded by Alt [1921], who managed to reduce the synthesis of transmission four-link mechanisms to the task of determining circular points. The next development of the theory of Burmester [1888] and the development of graphic methods for the synthesis of mechanisms based on this theory is connected with the developments of many German scientists, which are reflected in the summarized monographs of Bayer [1953,1959,1963], Hain [1967], Lichtenheld [1964]. Graphical solutions of the mechanism synthesis task lead to time-consuming graphical procedures and do not always ensure accuracy in reproducing the specified displacements. Such difficulties existed until the moment after which computer technology entered the graphics technologies massively, based on software products such as MathCAD, AutoCAD, Mat LAB, etc. However, these difficulties are one of the reasons for the evolution of the kinematic synthesis of mechanisms, in the direction of the development of the analytical methods of the extremely remote positions of a rigid body performing planar motion.

In this work, a practical method for drawing KKT and KCT is proposed, based on the condition of geometric constraint between a point of KKT and a point of KCT. Because this geometric constraint condition is mostly used in the vector-matrix method of lever mechanism synthesis, and the proposed method can be implemented in a software product environment with powerful computing and graphics capabilities, we name it the computerized vector-matrix method.

II. MATERIALS AND METHODS

II.1 *Conditions and algorithm for plotting Burmester's circle point curves and center point curves.*

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For a rigid body (most often a unit of a mechanism) that performs a general planar movement, the coordinates X_{ci} and Y_{ci} (i = 1,.....4) are set (or can be determined in some way) for four discrete positions of point **C** from the body and its three relative angular orientations φ_{1i} (i= 2, 3, 4).

To be drawn: the circle point curve (CPC) which represents the GMTs which for the four discrete positions of the body will lie on circles of a fixed plane and the center point curve (CCT) which represents the GMTs which are the centers of these circles.

The algorithm for solving the task set in this way has the following form:

1. It is denoted by $\mathbf{B_1}$ point of (KKT). The index of this notation indicates that it is a point of the moving plane at the first discrete position of the body;

2. It is denoted by **A** a point of (KCT), which is the center of a circle lying on the stationary plane and determined by the four discrete positions of the so-called **B**;

3. Using the plane displacement matrix, express the coordinates $\mathbf{X_{Bi}}$ and $\mathbf{Y_{Bi}}$ (i=2,3,4) of the discrete positions of point **B** in terms of the coordinates $\mathbf{X_{B1}}$ and $\mathbf{Y_{B1}}$ of the same point.

$$(1)\ \begin{bmatrix} X_{Bi} \\ Y_{Bi} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_{1i} & -\sin\varphi_{1i} & X_{Ci} - X_{C_1}\cos\varphi_{1i} + Y_{C_1}\sin\varphi_{1i} \\ \sin\varphi_{1i} & \cos\varphi_{1i} & Y_{C_1} - X_{C_1}\sin\varphi_{1i} - Y_{C_1}\cos\varphi_{1i} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{B_1} \\ Y_{B_1} \\ 1 \end{bmatrix}$$

4. The equations for the geometric constraint of the segment $\overline{AB}$ are written down.

$$(2)\quad a_i X_A + b_i Y_A = c_i \quad i=2,3,4\ , \quad \text{where}$$

$$(3)\quad \begin{aligned} a_i &= 2(X_{Bi} - X_{B_1}), \\ b_i &= 2(Y_{Bi} - Y_{B_1}), \\ c_i &= X_{Bi}^2 + Y_{Bi}^2 - X_{B_1}^2 - Y_{B_1}^2\,; \end{aligned}$$

5. The coordinates are also accepted for variable parameters $\mathbf{X_{B1}}$, $\mathbf{Y_{B1}}$ and the expression for the function of these parameters is recorded: Marinov [2002].

$$(4)\quad K = \frac{(c_2.b_3 - c_3.b_2)}{(a_2.b_3 - a_3.b_2)} - \frac{(c_2 b_4 - c_4.b_2)}{(a_2.b_4 - a_4 b_2)}\,.$$

6. An isoline is drawn on the surface determined by the function (4) for zero values of this function in the area determined by the limits of the variable parameters (min $X_{B_1} \leqslant X_{B_1} \leqslant \max X_{B_1}$) , (min $Y_{B_1} \leqslant Y_{B_1} \leqslant \max Y_{B_1}$). The dashed isoline is the CCP in this area.

$$(5)\quad \begin{bmatrix} X_{A'i} \\ Y_{A'i} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi_{1i} & \sin\varphi_{1i} & X_{Ci} - X_{C_1}\cos\varphi_{1i} - Y_{C_1}\sin\varphi_{1i} \\ -\sin\varphi_{1i} & \cos\varphi_{1i} & Y_{C_1} + X_{C_1}\sin\varphi_{1i} - Y_{C_1}\sin\varphi_{1i} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{A_1} \\ Y_{A_1} \\ 1 \end{bmatrix}$$

7. A kinematic inversion is performed (the moving plane becomes stationary, and the stationary plane becomes mobile). Using the inverse plane displacement matrix, the coordinates $X_{A'_i}$, $Y_{A'_i}$ of the inverse positions of point **A** (i=2,3,4) are also expressed through the coordinates $\mathbf{X_{A1}}$ and $\mathbf{Y_{A1}}$ of the same point.

8. The equations for the geometric constraint of the inverse position of the segment $\overline{AB}$ are recorded.

$$(6)\quad a'_i.X_{B_1} + b'_i Y_{B_1} = c'_i \qquad i=2,3,4, \qquad \text{where}$$

$$(7)\quad \begin{aligned} a'_i &= 2(X_{A'_i} - X_{A_1}), \\ b'_i &= 2(Y_{A'_i} - Y_{A_1}), \\ c'_i &= X^2{}_{A'_i} + Y^2{}_{A'_i} - X^2{}_{A_1} - Y^2{}_{A_1}\,; \end{aligned}$$

9. The coordinates X_{A_1} and Y_{A_1} are accepted for variable parameters and the expression for the function of these parameters is written Marinov [2002].

$$(8)\quad Q = \frac{(c'_2.b'_3 - c'_3.b'_2)}{(a'_2.b'_3 - a'_3.b'_2)} - \frac{(c'_2 b'_4 - c'_4.b'_2)}{(a'_2.b'_4 - a'_4 b'_2)}$$

10. The isoline of the surface determined by the function (8) is drawn for zero values of this function, in the areas determined by the limits of the variable parameters (min $X_{A_1} \leqslant X_{A_1} \leqslant \max X_{A_1}$) and (min $Y_{A_1} \leqslant Y_{A_1} \leqslant \max Y_{A_1}$). The dashed isoline is the CCT in this area.

11. If a circular point of the CCT is selected using **(2)**, the corresponding center point of the CCT is determined. When the center point is selected, by substituting in **(6)** the corresponding circular point is determined.

III. RESULTS AND DISCUSSION

III.1. *Computer solution*

Example: To draw the CCT and CCT of a link of a mechanism that performs general planar movement at given: X_{C_1} =150mm, Y_{C_1} =90mm, X_{C_2} =100mm, Y_{C_2} =70mm, X_{C_3} =50 mm, Y_{C_3} =40mm, X_{C_4} =20mm, Y_{C_4} = –20mm, φ_{12} =30^0, =70^0, =120^0.

The algorithm compiled above can only be implemented by means of an appropriately constructed program module, in the environment of a modern software product. For this purpose, the author compiled such a product in the environment of Mat LAB R2021b for Windows.

Through the compiled algorithm in the operating environment of MatLAB R2021b for Windows, calculations were performed and the curve of circular points (CCP) was drawn in an area with limits - 0.2m⩽ X_{B_1} ⩽0.4m; -0.2m⩽ Y_{B_1} ⩽0,4m depicted in Fig.1 and the curve of the center points (CCP) in an area with limits -0.2m⩽ X_{A_1} ⩽0.4m, -0.2m⩽ Y_{A_1} ⩽0.4m - respectively depicted in Fig.2.

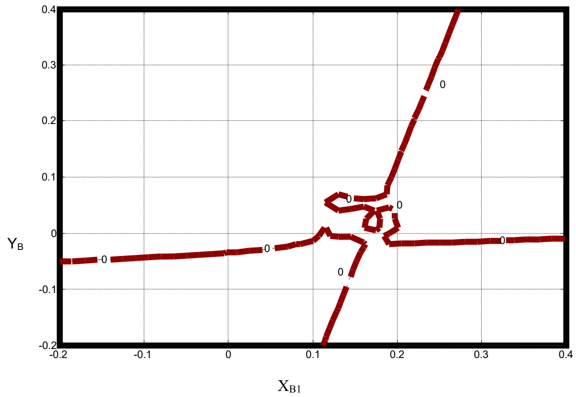

Fig.1 Burmester circle points curve

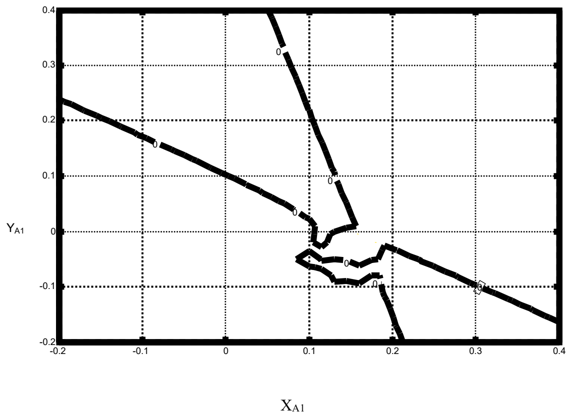

Fig.2 Burmester center points curve

IV. CONCLUSION

This article is intended for teaching bachelor's and master's students in the direction of mechanical engineering and the field of applied synthesis and kinematic analysis of mechanisms and machines for industry. The vector-matrix approach is applied to solving random problems from the theory of mechanisms and machines and automatic lines in industrial conditions. An example is shown through which a real solution to problems related to actuators and devices can be achieved.

A MatLAB R2021b for Windows program was used, with which different types of mechanisms can be synthesized, both in infinitely close and infinitely far positions of the input and output units. The classic Burmester principle was used as the basis of the synthesis, but adapted by the author for multilink mechanisms in industry [5].

Following the directions in which the synthesis of the mechanisms is developed on the basis of a relatively limited number of literary sources published in a relatively long period of time, the following classification of these directions can be proposed:

1. Kinematic geometry of finitely distant and infinitely close positions in planar motion of a rigid body.

2. Graphical synthesis of planar mechanisms in extremely distant and infinitely close positions, using the graphic methods of kinematic geometry in planar motion of a rigid body.

3. Analytical synthesis of planar mechanisms in extremely distant and infinitely close positions, using the analytical methods of kinematic geometry in planar motion of a rigid body.

4. Spatial kinematic geometry of extremely distant positions of a rigid body performing spatial motion.

5. Synthesis of spatial mechanisms in extremely distant positions, using the methods of spatial kinematic geometry.

6. Approximation synthesis of planar and spatial mechanisms, using quadratic, Chebyshevski and other approximations.

In order to carry out an optimal synthesis of an arbitrary mechanism with subsequent kinematic analysis and establishing the effect of the synthesis, the following activities were carried out:

1. In fig. 1 shows the isoline of the curve of Burmester circular points, the results obtained from a family of points so closely spaced as to form a curve which is characteristic of any synthesis of spatial or planar mechanism.

2. While fig. 2 shows the isoline of the curve of the Burmester center points, resulting in a family of points so closely spaced as to form a curve which is also characteristic of the synthesis of a spatial or planar mechanism.

3. The variable parameters are selected for Cartesian coordinates in the range: - 0.2m⩽ X_{B_1} ⩽0.4m; -0.2m⩽ Y_{B_1} ⩽0.4m for point B and -0.2m⩽ X_{A_1} ⩽0.4m, -0.2m ⩽ Y_{A_1} ⩽0.4m for point A .

4. Two or three calculation parameters are selected. The remaining two parameters are assumed to be variable at n=4 and n=5, or assigned appropriate values at n=3. A system of 2 (or 3) linear equations with unknown quantities - the calculation parameters - is compiled. The coefficients in front of the unknown quantities and the free terms are constants for n=3 or functions of the variable parameters for n=4 and n=5. At n=4, one function of the two variable parameters is compiled, and at n=5 - two such functions. These functions define a surface from the points of the isoline of the surface at n=4 with a zero value of the function defining the surface, the values of the variable parameters satisfying the synthesis conditions are taken into account. At n=5, the intersection points of the isolines of the surfaces of the two functions with zero values of these functions determine the variable parameters satisfying the synthesis conditions.

5. Variable parameters can also be selected in polar coordinates.

6. The author proposes a unified approach for the synthesis of circular point curves and center point curves using a computer and appropriate software, which can be applied to various types of transport robots and manipulators, both for moving and driving mechanisms.

7. Using the reported values of the variable parameters, the calculation parameters are determined.

8. For all compiled algorithms, synthesis program modules have been developed in the Math LAB for Windows environment.

9. The obtained numerical results were obtained theoretically and based on them a prototype was developed, which shows increased reliability when working in real conditions, compared to that of the existing models. The prototype of the mechanism is made in a real industrial environment.

The purpose of this publication is to raise the level of teaching in the field of the theory of machine mechanisms and automatic lines in industrial production.

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