Environment. Technology. Resources. Rezekne, Latvia
Proceedings of the 15th International Scientific and Practical Conference. Volume II, 297-302

Review of Methods for Classifying and Assessing Information Uncertainty

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Abstract. **There is a large number and variety of uncertainties that relate to almost all areas of life and the activity of people. Based on numerous literature sources, this article classifies various uncertainties into the following large groups: (1) stochastic uncertainties and (2) numerical uncertainties, including fuzzy and possibilistic ones. The article presents general approaches to assessing uncertainties in each of these groups. In addition, the article presents the main characteristics of general uncertainty estimates and provides an illustrative example.**

Keywords: uncertainty and information, statistical uncertainty, numerical uncertainty, estimation of uncertainty.

I. INTRODUCTION

The materials for research in this article are various publications on the topics presented. The article is a review nature. The presented methods are supplemented with illustrative examples.

The whole world is riddled with various kinds of uncertainties. What are the chances of rain tomorrow? How long will you have to wait for the tram at the stop? How old can a person be who is described as middle-aged? This kind of uncertainty can be continued indefinitely.

To clarify further considerations, we will introduce a generalized concept of a system. In the context of this article, we will consider information systems that consist of variables that represent one or another type of information (data). Let us quote from [1], which defines such systems and the potential uncertainties associated with them.

"In general, systems are viewed as relations among states of given variables. They are constructed for various purposes, such as prediction, retrodiction, extrapolation... control, planning, decision-making, scheduling, and diagnosis. In each system, its relation is utilized in each purposeful way to determine unknown states of some variables based on known states of some other variables.

Systems in which the unknown states are determined uniquely are called *deterministic systems*; all other systems are called *nondeterministic systems*. Each nondeterministic system involves uncertainty of some type. This uncertainty pertains to the purpose for which the system was constructed. It is thus natural to distinguish predictive uncertainty, retrodictive uncertainty... diagnostic uncertainty, and so on. In each nondeterministic system, the relevant uncertainty must be properly incorporated into the description of the system in some formalized language."

The key point in this quotation is that relevant uncertainties must be identified, displayed, and evaluated using appropriate formal language for expressing those uncertainties.

There is a close relationship between the amount of information and the uncertainty of this information. The absence or insufficiency of existing information usually causes uncertainty. In work [1] this connection is presented schematically (see Figure 1).

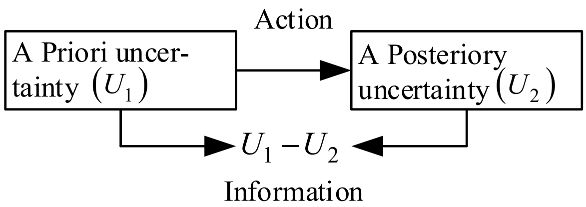

Fig. 1. Schematic representation of the relationship between uncertainty and information [1].

II. STATISTICAL UNCERTAINTIES

The best known are stochastic uncertainties. This is the field of probability theory and mathematical statistics. Uncertainties about the states of relevant variables are expressed by probabilistic estimates.

Let a set of random events $E = \{e_i / i = 1,...,n\}$ be given and the probabilities of the occurrence of each of

Print ISSN 1691-5402
Online ISSN 2256-070X
https://doi.org/10.17770/etr2024vol2.8024

these random events $\{p_i / i = 1,...,n\}$ estimated. Then the probabilistic estimates must satisfy the following axiomatic requirements.

1. $0 \le p_i \le 1, \forall i$.

2. If the events are independent of each other, then $p(e_1 \cup .... \cup e_n) = p(e_1) + ... + p(e_n)$.

3. If the set E is a complete group of random events, then $\sum_{i=1}^{n} p_i = 1$.

The concept of entropy as an estimate of the degree of stochastic uncertainty was introduced in [2]. Let a set of discrete values of a random variable X be given. Entropy as an estimate of the degree of uncertainty in the distribution of values X is calculated using the expression:

$$H(\mathbf{X}) = -\sum_{x_i \in \mathbf{X}} p(x_i) \log_2 p(x_i). \quad (1)$$

where $p(x_i)$ - the probability of occurrence of value x_i variable $\mathbf{X}$.

Let there be two sets of discrete values of random variables $\mathbf{X}$ and $\mathbf{Y}$ and the probabilities for the values of $x_i \in \mathbf{X}$ to occur depend on the values $y_j \in \mathbf{Y}$. Then conditional entropy for distribution $\mathbf{X}/\mathbf{Y}$ is defined as

$$H(\mathbf{X}/\mathbf{Y}) = -\sum_i \sum_j p(x_i / y_j) \log_2 p(x_i / y_j), \quad (2)$$

where $p(x_i / y_j)$ - the conditional probability of realization of value x_i subject to realization of value y_j.

If the discrete values of random variables $\mathbf{X}$ and $\mathbf{Y}$ are distributed independently, then joint entropy of their joint distribution is defined as

$$H(\mathbf{X},\mathbf{Y}) = -\sum_i \sum_j p(x_i, y_j) \log_2 p(x_i, y_j), \quad (3)$$

where $p(x_i, y_j)$ is the probability of joint realization of values x_i and y_j.

If the values of random variables $\mathbf{X}$ and $\mathbf{Y}$ are continuous, then the summation operations in expressions (1) – (3) are replaced by integration operations.

When calculating entropy, binary logarithms are usually used. The unit of entropy is then called a bit. In principle, logarithms with other bases can be used to estimate entropy. If logarithms with base e are used, then the unit of measurement is *nat*, if logarithms with base 10 are used, then the unit of measurement is *dit*.

In work [3] the following axiomatic characterization of the concept of entropy is given:

1. Subaddivity: for joint distributed random variables $H(\mathbf{X},\mathbf{Y}) \le H(\mathbf{X}) + H(\mathbf{Y})$.

2. Addivity: $H(\mathbf{X},\mathbf{Y}) = H(\mathbf{X}) + H(\mathbf{Y})$ when the random variables $\mathbf{X},\mathbf{Y}$ are independent.

3. Extensibility:

$H_{n+1}(p_1,...,p_n,0) = H_n(p_1,...,p_n)$ - adding an outcome with probability zero does not change the entropy.

4. Symmetry: $H_n(p_1,...,p_n)$ is invariant under permutation of $p_1,...,p_n$.

5. Small for small probabilities: $\lim_{q \to 0^+} H(1-q, q) \to 0$.

Entropy estimates are very widely used in various contexts to estimate the degree of uncertainty in statistical information.

III. Fuzzy entropy and Fuzziness estimates

Another large class consists of uncertainties associated with numerical estimates. Until the 60s of the 20th century, the only type of such uncertainties were measurement errors. The measured value was presented in the form $x \pm \alpha$, where α is the amplitude of possible measurement errors.

In 1965, L. A. Zadeh published his famous work [4], which laid the foundation for a new class of uncertainties - fuzzy sets and fuzzy numbers. Essentially, fuzzy numbers are fuzzy sets defined on some number axis.

The emergence of fuzzy set theory necessitated the development of new measures. The theory of fuzzy estimates was proposed in [5]. These measures, named after the author, are called fuzzy Sugeno measures. These estimates are determined as follows.

Let ρ be an σ-algebra on universe $\mathbf{X}$. A Sugeno fuzzy measure is $g: \rho \to [0,1]$, verifying:

1. $g(\varnothing) = 0$, $g(\mathbf{X}) = 1$.

2. If $A, B \in \rho$ and $A \subseteq B$, then $g(A) \le g(B)$.

3. If $A_n \notin \rho$ and $A_1 \subseteq A_2 \not\subset ...$ then $\lim_{n \to \infty} g(A_n) = g(\lim_{n \to \infty} A_n)$.

Property 2 is called monotony and property 3 is called Sugeno's convergence.

How can the degree of uncertainty associated with the original fuzzy information be measured? In this article, we will present three types of such measures.

Let a fuzzy random variable $\mathbf{X}$ be given, the values of which are triangular normal fuzzy numbers $\tilde{A}_j$. For clarity, an example of such a fuzzy number is graphically presented in Figure 2.

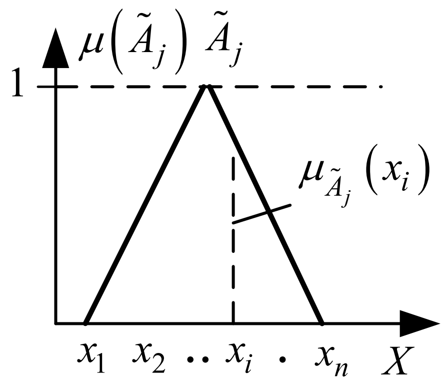

Fig. 2. Graphical representation of a triangular normal fuzzy number $\tilde{A}_j$.

To estimate the degree of uncertainty of a fuzzy random number, A. L. Zadeh in 1965 proposed the following extension of Shannon entropy

$$H(\tilde{A}_j) = -\sum_{i=1}^{n} \mu_{\tilde{A}_j}(x_i)\, p(x_i) \log_2 p(x_i), \quad (4)$$

where $\mu_{\tilde{A}_j}(x_j)$ is the value of the membership function of the element x_i to the fuzzy number $\tilde{A}_j$;

$p(x_i)$ - probability of element (value) x_i implementation.

Points $x_1, x_2, ..., x_n$ in Figure 2 represent the values of the fuzzy variable S in the interval reflecting the basis of the fuzzy number $\tilde{A}_j$. Unless otherwise assumed, the distribution of x_i values can be considered to be a uniform distribution. Then $p(x_i) = \frac{1}{n}$. The $\mu_{\tilde{A}}(x_i)$ values can be read from the graph or calculated if the analytical expression of function $\mu_{\tilde{A}}(x_i)$ is given.

If a fuzzy variable S includes m fuzzy numbers $\tilde{A}_j$, then the total fuzzy entropy for the variable S is defined as the sum of the fuzzy entropies of the fuzzy numbers forming it.

$$H(S) = \sum_{j=1}^{m} H(\tilde{A}_j). \quad (5)$$

In addition to estimates of fuzzy entropy for fuzzy random numbers, various estimates of the degree of fuzziness for fuzzy numbers have been proposed. It should be kept in mind that estimates of fuzzy entropy and estimates of the degree of fuzziness of fuzzy numbers are different estimates of the uncertainty of these fuzzy numbers. Fuzzy entropy estimates are analogous to Shannon entropy estimates in a fuzzy environment. Estimates of the degree of fuzziness are specific estimates of uncertainty, which is associated only with the forms of membership functions for these fuzzy numbers.

In this paper, we will present two common estimates of the degree of fuzziness for fuzzy numbers.

De Luca and Termini [6] proposed the following estimate of the degree of fuzziness of the fuzzy number $\tilde{A}$. To visualize further definitions, Figure 3 graphically represents the triangular normal fuzzy number $\tilde{A}$ and its complement $\bar{\tilde{A}}$.

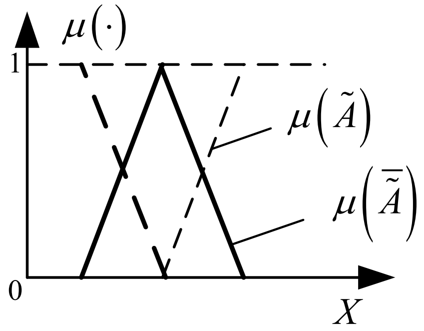

Fig. 3. Graphical representation of a triangular normal fuzzy number $\tilde{A}$ and its complement $\bar{\tilde{A}}$.

$$d(\tilde{A}_j) = H(\tilde{A}_j) + H(\bar{\tilde{A}}). \quad (6)$$

Using Shannon's function

$$S(x) = -x \ln x - (1-x)\ln(1-x), \quad (7)$$

expression (6) can be represented in the following form

$$d(\tilde{A}_j) = k\sum_{i=1}^{n} S(\mu_{\tilde{A}_j}(x_i)). \quad (8)$$

where k is a positive constant.

Note that the Shannon function (7) uses natural logarithms. This is not of fundamental importance. The choice of logarithm base only affects the units of measurement of the resulting uncertainty. If you want to express the estimated degree of fuzziness in bits, then use $\log_2(x)$ instead of $\ln(x)$.

Estimates of the degree of fuzziness De Luca and Termini must satisfy the following obvious requirements [7]:

1. $d(A) = 0$ if A is a crisp set in $\mathbf{X}$.
2. $d(\tilde{A})$ assumes a unique maximum if $\mu_{\tilde{A}}(x) = \frac{1}{2}, \forall x \in \mathbf{X}$.
3. $\mu_{\tilde{A}'}(x) \le \mu_{\tilde{A}}(x)$ if $\tilde{A}'$ is "crisper" then $\tilde{A}$, i. e., $\mu_{\tilde{A}'}(x) \le \mu_{\tilde{A}}(x)$ for $\mu_{\tilde{A}}(x) \le \frac{1}{2}$ and $\mu_{\tilde{A}'}(x) \ge \mu_{\tilde{A}}(x)$ for $\mu_{\tilde{A}}(x) \ge \frac{1}{2}$.
4. $d(\tilde{A}) = d(\bar{\tilde{A}})$ where $\bar{\tilde{A}}$ is complement of $\tilde{A}$.

Another assessment of the degree of fuzziness of a fuzzy number $\tilde{A}$ has the following basis. For a fuzzy number $\tilde{A}$ and its complement $\bar{\tilde{A}}$, in contrast to crisp sets, the following statements are not mandatory:

$$\tilde{A} \cup \bar{\tilde{A}} = \mathbf{X};$$

$$\tilde{A} \cap \bar{\tilde{A}} = \varnothing.$$

In work [8] R. Yager argues that the assessment of the degree of fuzziness for a fuzzy number $\tilde{A}$ should reflect the difference between this number and its complement $\bar{\tilde{A}}$. In other words, the estimate of the degree of fuzziness should be a function of the distance between $\tilde{A}$ and $\bar{\tilde{A}}$ or between $\mu_{\tilde{A}}(x)$ and $\mu_{\bar{\tilde{A}}}(x)$. In [8] the author proposed the following metric for estimating the distance between $\tilde{A}$ and $\bar{\tilde{A}}$:

$$D_p\left(\tilde{A}, \bar{\tilde{A}}\right) = \left[\sum_{i=1}^{n} \left|\mu_{\tilde{A}}(x_i) - \mu_{\bar{\tilde{A}}}(x_i)\right|^p\right]^{\frac{1}{p}}, p = 1, 2, ... \quad (9)$$

This is the Minkowski metric. For $p = 1$ we have the Hamming metric:

$$D_1\left(\tilde{A}, \bar{\tilde{A}}\right) = \sum_{i=1}^{n} \left|\mu_{\tilde{A}}(x_i) - \mu_{\bar{\tilde{A}}}(x_i)\right|. \quad (10)$$

If $\mu_{\bar{\tilde{A}}}(x) = 1 - \mu_{\tilde{A}}(x)$ then

$$D_1\left(\tilde{A}, \bar{\tilde{A}}\right) = \sum_{i=1}^{n} \left|2\mu_{\tilde{A}}(x_i) - 1\right|. \quad (11)$$

For $p = 2$ we have the Euclidian metric:

$$D_2\left(\tilde{A}, \bar{\tilde{A}}\right) = \left(\sum_{i=1}^{n} \left(\mu_{\tilde{A}}(x_i) - \mu_{\bar{\tilde{A}}}(x_i)\right)^2\right)^{\frac{1}{2}}. \quad (12)$$

If $\mu_{\bar{\tilde{A}}}(x) = 1 - \mu_{\tilde{A}}(x)$ then

$$D_2\left(\tilde{A}, \bar{\tilde{A}}\right) = \left(\sum_{i=1}^{n} \left(2\mu_{\tilde{A}}(x_k) - 1\right)^2\right)^{\frac{1}{2}}. \quad (13)$$

There are other definitions of the complement of a fuzzy set $\bar{\tilde{A}}$. In such cases, the values of $\mu_{\bar{\tilde{A}}}(x_i)$ should appear explicitly in Expressions (8), (10), which are calculated on the basis of the corresponding definition of $\bar{\tilde{A}}$.

Additional information about fuzzy entropy and fuzzy estimates can be found in the works [7], [8], [9].

To model uncertainties greater than those modeled by standard fuzzy numbers the following extensions of fuzzy numbers have been proposed:

1. Interval-valued fuzzy numbers [10].
2. Fuzzy numbers type-2 [11].
3. Interval-valued fuzzy numbers type-2 [12],[13].
4. Intuitionistic fuzzy numbers [14].

In this work, we limit ourselves to considering only standard fuzzy numbers.

Various approaches to combining statistical and fuzzy information have been proposed. In this sense, we can talk about the probabilities of fuzzy events [15] and fuzzy probability estimates.

IV. Possibility Theory and Hartley Measure

To model very high degrees of uncertainty, a possibility theory has been proposed [16], [17].

Assume that the universe of discourse Ω is a finite set. A *possibility measure* is a function Π from 2^{Ω} to [0,1], such that:

1. $\Pi(\varnothing) = 0$;
2. $\Pi(\Omega) = 1$;
3. $\Pi(U \cup V) = \max(\Pi(U), \Pi(V))$ for any disjoint subsets U and V.

Let there be a numerical or non-numerical set A, all of whose elements are equally possible. The degree of uncertainty of this set is estimated based on the Hartley function [18]

$$H_0(A) = \log_2 |A|. \quad (14)$$

where $|A|$ denotes the cardinality of the set A.

To better visualize the difference between fuzzy and possibilistic uncertainties, let's look at Figure 4 [19]. Figure 4 (a) shows a graph of the membership function of a triangular normal fuzzy number $\tilde{A}$.

For any value $x \in \tilde{A}$, the degree of its membership to $\tilde{A}$, $\mu_{\tilde{A}}(x)$, is uniquely determined (vertical arrows in Figure 4 (a)).

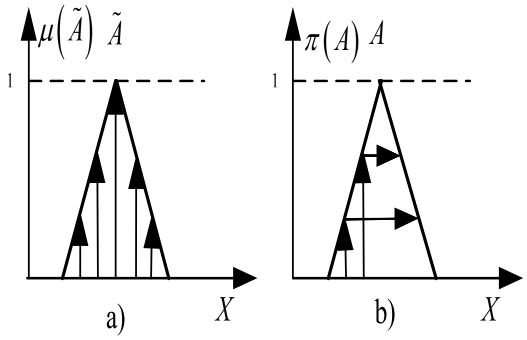

Fig. 4. Graphical representation of the triangular normal fuzzy number $\tilde{A}$ (a) and the possibilistic number A (b). [19]

Figure 4 (b) shows a graph of the distribution of possibilities for the possibilistic number A. For any value of $x \in A$, the degree of its possibility is determined by the horizontal segment of the line connecting the corresponding points on the graph $\pi(A)$ (horizontal lines in Figure 4 (b)).

This is the significant difference between fuzzy and possibilistic numbers, which are expressions of fundamentally different types of numerical uncertainties.

V. ILLUSTRATIVE EXAMPLE

In Figure 5 graphically shows the distribution of three fuzzy probabilities $\tilde{P}=\left(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3\right)$.

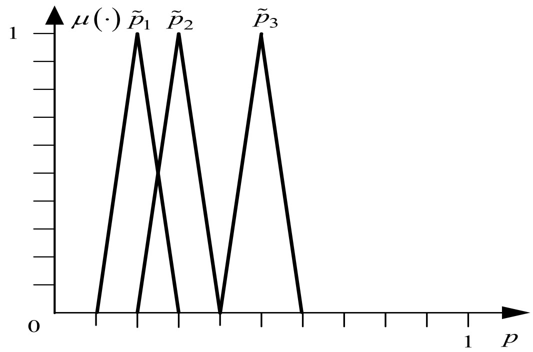

Fig. 5. Graphical representation of the distribution of three fuzzy probabilities.

Required:

1. Calculate the fuzzy entropy value of this distribution.

2. Calculate estimates of the degree of fuzziness of fuzzy numbers $\tilde{p}_1,\tilde{p}_2,\tilde{p}_3$ using the expressions (11), (13).

Using the graph of the membership function of a fuzzy number $\mu\left(\tilde{p}_1\right)$, we determine the following reference values of probability p_1, the corresponding values of the membership function $\mu\left(p_1\right)$ and the probability of the implementation $p^*\left(p_1\right)$ of these reference values:

p_1	0.10	0.15	0.20	0.25	0.30.
$\mu\left(p_1\right)$	0.00	0.50	1.00	0.50	0.00.
$p^*\left(p_1\right)$	0.20	0.20	0.20	0.20	0.20.

The values of $p^*\left(p_1\right)$ are assigned from the statement that the values of p_1 are uniformly distributed on the base of a fuzzy number $\tilde{p}_1$.

According to expression (4):

$$\begin{aligned}
&H\left(\tilde{p}_1\right)=-(0.00*0.20*\left(-0.699\right)+\\
&+0.50*0.20*\left(-0.699\right)+\\
&+1.00*0.20*\left(-0.699\right)+\\
&+0.50*0.20*\left(-.0699\right)+\\
&+0.00*0.20*\left(-0.699\right))=\\
&=0.000+0.070+0.140+0.070+0.000=\\
&=0.280.
\end{aligned}$$

Since the shapes of the graphs of the membership functions of the fuzzy probabilities $\tilde{p}_2,\tilde{p}_3$ are exactly the same as for the fuzzy probability $\tilde{p}_1$, the fuzzy entropy estimates for all three fuzzy numbers are the same. Therefore, the value of fuzzy entropy for distribution $\tilde{P}$ based on the property of additivity of fuzzy entropy estimates is equal to:

$$H\left(\tilde{P}\right)=3*0.280=0.840\ .$$

According to expression (11):

$$\begin{aligned}
&D\left(\tilde{p}_1,\bar{\tilde{p}}\right)=\left|2*0.00-1\right|+\left|2*0.50-1\right|+\\
&+\left|2*1.00-1\right|+\left|2*0.50-1\right|+\left|2*0.00-1\right|=\\
&=1.00+0.00+1.00+0.00+1.00=3.00.
\end{aligned}$$

According to expression (13):

$$\begin{aligned}
&D_2\left(\tilde{p}_1,\bar{\tilde{p}}\right)=((2*0.00-1)^2+\left(2*0.50-1\right)^2+\\
&+(2*1.00-1)^2+\left(2*0.50-1\right)^2+\\
&+\left(2*0.00-1\right)^2)^{\frac{1}{2}}=\\
&=\left(1.00+0.00+1.00+0.00+1.00\right)^{\frac{1}{2}}=3^{\frac{1}{2}}=1.732.
\end{aligned}$$

VI. CONCLUSIONS

This article provides a brief overview of the most common types of uncertainty in data. Some uncertainties can manifest themselves as phenomena of the external world (statistical uncertainties). Another type of uncertainty is a consequence of the lack of clear boundaries between sets (numbers).

These types of uncertainties are usually identified as fuzzy uncertainties. Possibilistic uncertainties are an extreme type of uncertainty when all elements of a certain set or numerical values in a given interval are equally possible.

Identifying existing uncertainties in data is very important for processing and analyzing these data. In some circumstances it is necessary to estimate the degree of uncertainty in existing data. This article presents the main approaches to assessing various types of uncertainties. Definitions of fuzzy entropy and degree of fuzziness are presented only for triangular normal fuzzy numbers. It should be noted that expressions for calculating fuzzy entropy and degree of fuzziness are proposed for all fuzzy number extensions mentioned in this paper, as well as for other less common fuzzy number extensions.

Fuzzy entropy's estimates are widely used to solve various kinds of practical problems. Works [20], [21] present examples of using fuzzy entropy estimates in decision making problems. Works [22], [23] present examples of using fuzzy entropy estimates in data analysis problems. In work [24], fuzzy entropy estimates are used to solve the supplier selection problem. Works [25] – [27] present the use of fuzzy entropy in classification and clustering problems.

The general conclusion from this article: in order to correctly use uncertain data (information), it is necessary to clearly establish the nature of the existing uncertainty and use methods suitable for this type of identified uncertainty.

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