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Characteristics of Motions in Facing

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Abstract. **In the process of work the tool angles change their value to become the working angles. This results in a variation of the work conditions of the cutting tools. The variation depends on the characteristic of the resultant cutting motion – the sum of the cutting motion and feed motion.**

In this article the characteristics of motions of facing are derived, which can serve as a basis for determining the relations between the tool and working angles.

These relations can be used in programming the CNC to prevent the formation of negative working clearance angle and the destruction of the tool.

Keywords: characteristics of movement, coordinate axes, cutting and feed motion, resultant cutting motion.

I. INTRODUCTION

The ISO 3002-1982 standard helps to solve many engineering tasks, but it is not generally valid and computer oriented. The simultaneous existence of two standards - ASME B94.50-1975 and ISO3002-1:1984, which deal with unchangeable terminology and definitions, has a large share in the confusion and misunderstanding of the basic geometric parameters of cutting tools [1].

A critical analysis of both the advantages and disadvantages of ISO 3002-1:1984 can be found in several sources [1] - [6].

In the alternative methodology of [7], proposed in [2], the dependencies between tool and working angles are derived by defining in a new way the setting angles *G*, *H* and *L* that give the correlation between the tool-in-hand system *f* and the machine system *m,* and the motion angles *M*, *N* and *T* that connect the tool-in-use system f_e with the machine system *m*.

The variation of tool angles in working ones in the process of operation is due to the feed rate D_f. The feed rate D_f, summed with the cutting motion D_c, determines the resultant cutting motion D_e. The placement of the tool in relation to the workpiece also has an impact.

For transformations of the tool angles into working ones and vice versa it is necessary that the main characteristics of the cutting motion D_c, feed rate D_f and the resultant cutting motion D_e: trajectory, path, speed, acceleration are known. The local elements of the trajectory: the tangent, the principal normal, the binormal (which form an accompanying trihedron), curvature and torsion should be known as well.

For the practical application of these dependencies the parametric equations of resultant cutting motion in facing have been derived in [8]. In this article the equations of the characteristics of the cutting, feed and resultant cutting motions are derived.

II. METHODS

A. Equations for the resultant cutting motion and the motions of cut and feed.

Two rectangular right oriented coordinate systems, fixed to the workpiece $O_{1X_1Y_1Z_1}$ and the machine $O_{0X_0Y_0Z_0}$, respectively, were introduced. Axes X_0, Y_0 and Z_0 are colinear with the axes of a standard coordinate system as in [9].

The case of cutting motion $-C_c'$ and feed motion $-X_f$ is considered when facing from the center to the periphery (Fig. 1) and $+X_f$ when facing from the periphery to the center (Fig. 2).

Depending on the placement of the tool relative to the center axis of the machine, three cases are possible [8] – position above the center axis (angle *N*>0) as in Fig. 1 (a) and Fig. 2 (a); position on the axis of the centers (angle *N*=0) as in Fig. 1 (b) and Fig. 2 (b); position below the axis of the centers (angle *N*<0) as in Fig. 1 (c) and Fig. 2 (c).

The resultant cutting motion for a point of the cutting edge of a tool with coordinates *x = x(t)*, *y = const.* and *z = const.* is defined in $O_{0X_0Y_0Z_0}$ with the equation

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$$\overline{r}_e(t)=\begin{vmatrix} x_e(t)\\ y_e(t)\\ z_e\end{vmatrix}=\begin{vmatrix} r(t).\cos\left[N-\varphi_{01}(t)\right]\\ r(t).\sin\left[N-\varphi_{01}(t)\right]\\ z\end{vmatrix}, \qquad (1)$$

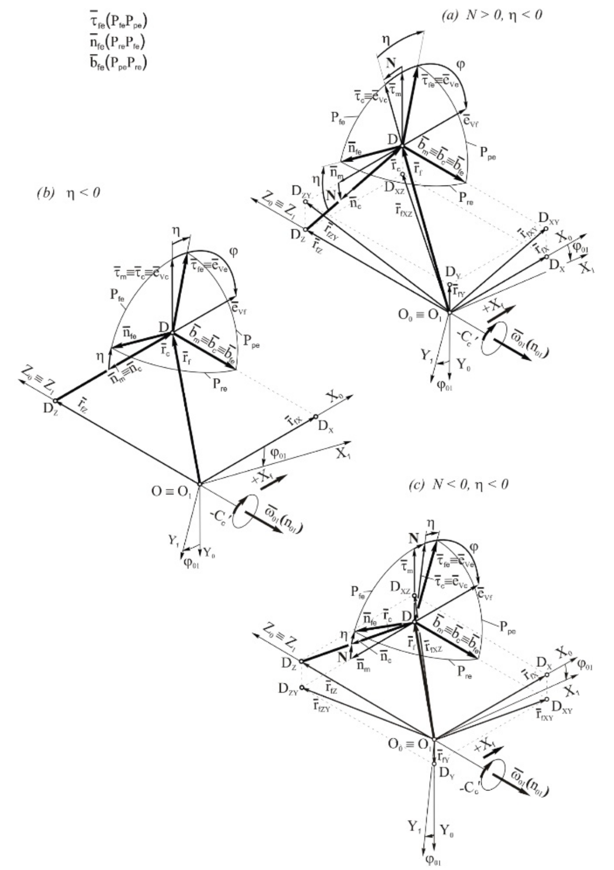

Fig. 1 Tool-in-use system f_e ($\overline{\tau}_{fe}$, $\overline{n}_{fe}$, $\overline{b}_{fe}$) with cutting motion $-C'_c$ and feed motion $+X_f$.

where φ_{01} is the angle of rotation around $Z_0 \equiv Z_1$ of the workpiece coordinate system $O_{1X_1Y_1Z_1}$ against coordinate system of the machine $O_{0X_0Y_0Z_0}$, and

$$r(t)=\sqrt{x(t)^2+y^2}=\sqrt{\left(X+i.n.f.t\right)^2+y^2} \qquad (2)$$

$$x(t)=X+i.n.f.t\,. \qquad (3)$$

The tool-in-use system f_e [8] is oriented in the machine system m (Fig.1, Fig.2) according to the cutting direction giving angle N

$$N(t)=arctg\left(-\frac{y}{x}\right)=arctg\left(-\frac{y}{X+i.n.f.t}\right). \qquad (4)$$

(1) is an equation of a flat curve, and when $y = 0$ – archimedean spiral.

The feed motion $-Z_f$ in $O_{0X_0Y_0Z_0}$ is described by the equation:

$$\overline{r}_f(t)=\begin{vmatrix} x_f(t)\\ y\\ z\end{vmatrix}=\begin{vmatrix} X+i.n.f.t\\ y\\ z\end{vmatrix}. \qquad (5)$$

(5) is an equation of straight line perpendicular to the axis $Z_0 \equiv Z_1$ and lying in the plane $Y = y$. When the feed is $-X_f$, $i = -1$. When the feed is $+X_f$, $i = 1$.

The cutting motion is dependent on the feed motion and its characteristics are determined at a cutting time point t.

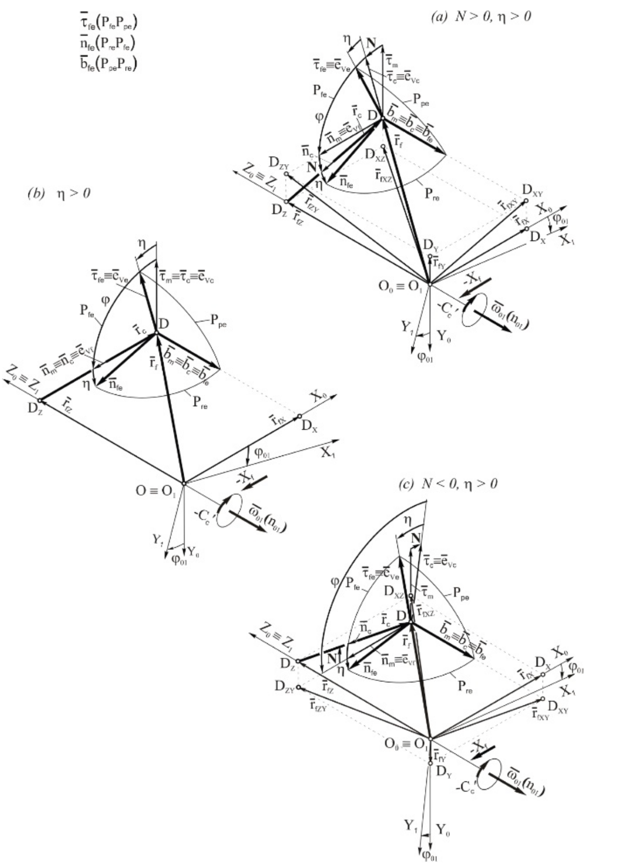

Fig. 2. Tool-in-use system f_e ($\overline{\tau}_{fe}$, $\overline{n}_{fe}$, $\overline{b}_{fe}$) with cutting motion $-C'_c$ and feed motion $-X_f$.

When both the revolutions n, min^{-1}, and the feed f, mm/min, are constant, the values of the angular speed ω and the feed speed v_f are

$$\omega = 2.\pi.n \qquad (6)$$

$$v_f = f.n. \qquad (7)$$

In this case the angle of rotation is

$$\varphi_{01} = \omega.t = 2.\ \pi.n.t, \qquad (8)$$

and the translation along the direction of feed is:

$$x(t) = V_f.t = f.\ n.t. \qquad (9)$$

Then (1) gets the form

$$\overline{r}_e(t)=\begin{vmatrix} x_e(t)\\ y_e(t)\\ z_e\end{vmatrix}= =\begin{vmatrix} \sqrt{\left(X+i.n.f.t\right)^2+y^2}.\cos\left[N-2.\pi.n.t\right]\\ \sqrt{\left(X+i.n.f.t\right)^2+y^2}.\sin\left[N-2.\pi.n.t\right]\\ z\end{vmatrix}. \qquad (10)$$

B. Characteristics of cut motion.

Value v_c of cutting speed $\overline{v}_c$ at a point in time t is:

$$v_c(t)=\omega.r(t)=2.\pi.n.\sqrt{\left(f.i.n.t+X\right)^2+y^2}\;. \quad (11)$$

C. Characteristics of feed motion.

The feed speed $\overline{v}_f$ is:

$$\overline{v}_f(t)=\left[i.n.f,0,0\right]^T, \quad (12)$$

and its value v_f is:

$$v_f=f.n=const. \quad (13)$$

The unit vector $\overline{e}_{v_f}$ of the feed speed $\overline{v}_f$ is:

$$\overline{e}_{v_f}=\left[i,0,0\right]^T. \quad (14)$$

D. Characteristics of resultant cutting motion.

The speed $\overline{v}_e$ is:

$$\overline{v}_e(t)=\left\{v_{ex_1}(t),v_{ey_1}(t),v_{ez_1}(t)\right\}^T= \quad (15)$$

$$=\left|\begin{matrix}\dfrac{n.\left[f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)-A.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}}\\ -\dfrac{n.\left[A.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.}\\ 0\end{matrix}\right|,$$

where:

$$A=2.f^2.n^2.\pi.t^2+f.i.\left(4.n.\pi.t.X-y\right)+2.\pi.\left(X^2+y^2\right). \quad (16)$$

The value v_e is:

$$v_e(t)=n.\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}\;. \quad (17)$$

The unit vector $\overline{e}_{v_e}$ for the speed $\overline{v}_e$ is:

$$\overline{e}_{v_e}(t)=\frac{\overline{v}_e(t)}{v_e}= \quad (18)$$

$$=\left|\begin{matrix}\dfrac{f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)-A.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}}\\ -\dfrac{A.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}}\\ 0\end{matrix}\right|,$$

where A is defined by (16).

The acceleration $\overline{a}_e$ is:

$$\overline{a}_e(t)=\left|\begin{matrix}a_{ex_1}(t)\\ a_{ey_1}(t)\\ a_{ez_1}(t)\end{matrix}\right|= \quad (19)$$

$$=\left|\begin{matrix}-\dfrac{4.n^2.\pi.\left[B.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}}\\ \dfrac{4.n^2.\pi.\left[-f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+B.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}}\\ 0\end{matrix}\right|,$$

where:

$$B=f^2.n^2.\pi.t^2+f.i.\left(2.n.\pi.t.X-y\right)+\pi.\left(X^2+y^2\right). \quad (20)$$

The value a_e is:

$$a_e(t)=\sqrt{a_{ex_1}^2+a_{ey_1}^2+a_{ez_1}^2}= \quad (21)$$

$$=4.n^2.\pi.\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}\;.$$

The unit vector $\overline{e}_{a_e}$ of acceleration $\overline{a}_e$ is:

$$\overline{e}_{a_e}(t)=\frac{\overline{a}_e(t)}{a_e}= \quad (22)$$

$$=\left|\begin{matrix}-\dfrac{B.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}}\\ \dfrac{-f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+B.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}}\\ 0\end{matrix}\right|,$$

where B is defined by (20).

The value $a_{e\tau}$ of the tangential acceleration $\overline{a}_{e\tau}$ is:

$$a_{e\tau}(t)=\frac{\overline{v}_e.\overline{a}_e}{v_e}=\frac{v_{ex}.a_{ex}+v_{ey}.a_{ey}+v_{ez}.a_{ez}}{\sqrt{v_{ex}^2+v_{ey}^2+v_{ez}^2}}= \quad (23)$$

$$=\frac{4.f.n^2.\pi^2.\left(f.n.t+i.X\right)}{\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}},$$

and the value a_{en} of the normal acceleration $\overline{a}_{en}$ is:

$$a_{en}(t)=\sqrt{a_e^2-a_{e\tau}^2}= \quad (24)$$

$$=\frac{\sqrt{\left(v_{ex}.a_{ey}-a_{ex}.v_{ey}\right)^2+\left(v_{ey}.a_{ez}-a_{ey}.v_{az}\right)^2+\left(v_{ez}.a_{ex}-a_{ez}.v_{ex}\right)^2}}{\sqrt{v_{ex}^2+v_{ey}^2+v_{ez}^2}}=$$

$$=4.n^2.\pi.\sqrt{\begin{matrix}f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)-\\ -\dfrac{f^2.\pi^2.\left(f.i.n.t+X\right)^2}{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}\end{matrix}}$$

Radius $R_{k_{1e}}$ of the curve k_{1e} of the trajectory of resultant cutting motion is:

$$R_{k_{1e}}=\frac{1}{k_{1e}}=\frac{v_e^2}{a_{en}}= \quad (25)$$

$$=\frac{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}{4.\pi.\sqrt{\dfrac{\left[f^2.\left(1+2.n^2.\pi^2.t^2\right)+f.i.\pi.\left(4.n.\pi.t.X-3.y\right)+2.\pi^2.\left(X^2+y^2\right)\right]^2}{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}}}$$

The unit vector $\overline{\tau}_{fe}$ of tangent of the trajectory of resultant cutting motion (Fig.1, Fig.2) is:

$$\overline{\tau}_{fe}(t)=\overline{e}_{v_e}(t)= \quad (26)$$

$$= \left| \begin{matrix} -\dfrac{f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)-A.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}} \\ -\dfrac{A.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)}{\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right)}} \\ 0 \end{matrix} \right|,$$

where A is defined by (16).

The unit vector $\overline{n}_{fe}$ of the principal normal of the trajectory of resultant cutting motion is (Fig.1, Fig.2):

$$\overline{n}_{fe}(t) = \left| \begin{matrix} a_{ex}\left(v_{ey}^2+v_{ez}^2\right)-v_{ex}\left(v_{ey}.a_{ey}+v_{ez}.a_{ez}\right) \\ a_{ey}\left(v_{ex}^2+v_{ez}^2\right)-v_{ey}\left(v_{ex}.a_{ex}+v_{ez}.a_{ez}\right) \\ a_{ez}\left(v_{ex}^2+v_{ey}^2\right)-v_{ez}\left(v_{ex}.a_{ex}+v_{ey}.a_{ey}\right) \end{matrix} \right| = \qquad (27)$$

$$= \left| \begin{matrix} \dfrac{-C.\left[D.\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+f.i.\left(f.i.n.t+X\right).\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{E.\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}} \\ \dfrac{C.\left[-f.i.\left(f.i.n.t+X\right).\cos\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)+D.\sin\left(2.n.\pi.t+\arctan\dfrac{y}{f.i.n.t+X}\right)\right]}{E.\sqrt{\left(f.i.n.t+X\right)^2+y^2}.\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}} \\ 0 \end{matrix} \right|,$$

where:

$$C = f^2.\left(1+2.n^2.\pi^2.t^2\right)+f.i.\pi.\left(4.n.\pi.t.X-3.y\right)+2.\pi^2.\left(X^2+y^2\right), \qquad (28)$$

$$D = 2.f^2.n^2.\pi.t^2+f.i.\left(4.n.\pi.t.X-y\right)+2.\pi.\left(X^2+y^2\right), \qquad (29)$$

$$E = f^2.\left(1+4.n^2.\pi^2.t^2\right)+4.f.i.\pi.\left(2.n.\pi.t.X-y\right)+4.\pi^2.\left(X^2+y^2\right). \qquad (30)$$

The unit vector $\overline{b}_{fe}$ of binormal of the trajectory of resultant cutting motion (Fig.1, Fig.2) is:

$$\overline{b}_{fe}(t) = \frac{\overline{v}_e \times \overline{a}_e}{\left|\overline{v}_e \times \overline{a}_e\right|} = \left| \begin{matrix} v_{ey}.a_{ez}-a_{ey}.v_{ez} \\ v_{ez}.a_{ex}-a_{ez}.v_{ex} \\ v_{ex}.a_{ey}-a_{ex}.v_{ey} \end{matrix} \right| = \qquad (31)$$

$$= \left| \begin{matrix} 0 \\ 0 \\ \dfrac{-f^2.\left(1+2.n^2.\pi^2.t^2\right)+f.i.\pi.\left(-4.n.\pi.t.X+3.y\right)-2.\pi^2.\left(X^2+y^2\right)}{\sqrt{f^2.\left(1+n^2.\pi^2.t^2\right)+2.f.i.\pi.\left(n.\pi.t.X-y\right)+\pi^2.\left(X^2+y^2\right)}.\sqrt{E}} \end{matrix} \right|,$$

where E is defined by (30).

III. Conclusion

The change of tool angles in some working cases (turning acme threads, face turning, milling of complex profile surfaces, high speed cutting) leads to significant changes in the values of clearance angles, sometimes leading to tool destruction. The coordinate systems, setting angles and motion angles, defined in ISO 3002/2-1984(E), lead to complicated, unclear and in some cases impossible transformations of tool angles to working angles.

The derived equations for the characteristics of the cutting, feed and resultant cutting motions are the basis for deriving the dependencies for straight and inverse transformation between tool and working angles when facing.

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