SIMPLE METHODS OF ENGINEERING CALCULATION FOR SOLVING STATIONARY 2 –D HEAT TRANSFER PROBLEMS IN MULTILAYER MEDIA

Vienkāršas inženiertehniskas formulas stacionāras 2-D siltuma vadīšanas problēmas risināšanai daudzslāņu vidēs

H.Kalis, I.Kangro[1]
Institute of Mathematics Latvian Academy of Sciences and University of Latvia
Akadēmijas laukums1, LV – 1524, Rīga, Latvija, E-mail:kalis@lanet.lv
[1]Rēzekne Higher Education Institution, Department of engineering Science
Atbrīvošanas aleja 90, LV – 4601, Rēzekne, Latvija, E-mail:kangro@cs.ru.lv

Abstract
There are well-known different numerical methods for solving the boundary value problems for partial differential equations. Some of them are: finite difference method (FDM), finite element method (FEM), boundary element methods (BEM), and others. In the given work two methods FDM and BEM for the mathematical model of stationary distribution of heat in the multilayer media are considered. These methods were used for the reduction of the two-dimensional heat transfer problem described by a partial differential equation to a boundary – value problem for a system of ordinary differential equations. (ODEs). Such a procedure allows obtaining simple engineering algorithms for solving heat transfer equation in mulyilayer domain. In the case of three layers the system of ODEs is possible for solving analytically..
Keywords: *Poisson's type partial differential equation, two dimensional heat transfer problem.*

Mathematical model

We will consider the partial differential equation of Poisso`n type

$$\partial(\lambda\partial u/\partial y)/\partial y+\partial(\lambda\partial u/\partial z)/\partial z=-q(z,y) \qquad (1.1),$$

where λ is the coefficient of heat conductivity $(W/m\cdot K)$, q is the function of the thermal sources (W/m^3), (z,y) are the 2 – D space coordinates in the space (m), $u=u(z,y)$ is the absolute temperature (K). Multilayer media Ω in the z - direction, which consists of N layers $\Omega=\{(z,y):z\in\Omega_k, k=\overline{1,N}; y\in[0,L]\}$, where each layer is characterized by set $\Omega=\{(z,y):z_{k-1}\le z\le z_k, k=\overline{1,N}; y\in[0,L]\}$, and $z=z_k$, $k=\overline{1,N-1}$ are the joint of the layers (the interior grid points in the FDM), $y=0, y=L, z=0, z=z_N$ are the surfaces on the rectangle domain.

If every layer has parameters of λ_k, q_k, then the equation (1.1) can be presented in the form of $\partial(\lambda_k\partial u_k/\partial z)/\partial z=F_k$, $k=\overline{1,N}$, (1.2)

where $F_k=-\partial(\lambda\partial u/\partial y)/\partial y-q_k(z,y)$, $u_k=u_k(z,y)$ is the temperature in the layer Ω_k.

We have the following conditions:

1) conditions of a continuity on interior surfaces $z=z_k, k=\overline{1,N-1}$:

$$u_k(z_k,y)=u_{k+1}(z_k,y),\ \ \lambda_k\partial u_k(z_k,y)/\partial z=\lambda_{k+1}\partial u_{k+1}(z_k,y)/\partial z,$$

2) boundary conditions on the surfaces $z=0$ and $z=z_N$ in the form of

$$\begin{cases}\lambda_1\partial u_1(z_0,y)/\partial z-\alpha_1(u_1(z_0,y)-\theta_0)=0\\ \lambda_N\partial u_1(z_N,y)/\partial-\alpha_N(u_N(z_N,y)-\theta_N)=0\end{cases}, \qquad (1.3)$$

where α_1,α_N are convective heat transfer coefficients, $\theta_{0,}\theta_N$, $u^{(0)}(z)$ are the dimensionless temperatures of air.

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3) boundary conditions on the surfaces $y=0$ and $y=L$ in the form
$\partial u(z,0)/\partial y=0$ - the symmetry condition, $u(z,L)=u^{(0)}(z)$ - the given temperature or $\partial u(z,L)/\partial y=0$ - in the 1-D case.

Method of finite volumes and fdm

Using a method of finite volumes [1-4], we would find the finite- difference scheme of $N+1$ equations on a joint of layers in the form

$$\begin{cases} \lambda_1/h_1(u_1-u_0)-\alpha_1(u_0-\theta_0)=R_0^+ \\ \lambda_{k+1}/h_{k+1}(u_{k+1}-u_k)-\lambda_k/h_k(u_k-\theta_{k-1})=R_k^-+R_k^+ \\ -\alpha_N(u_N-\theta_N)-\lambda_N/h_N(u_N-u_{N-1})=R_N^-, \end{cases} \tag{2.1}$$

where $k=\overline{1,N-1}$, $R_k^-=I_k^-+\tilde{R}_k^-$, $R_k^+=I_k^++\tilde{R}_k^+$,

$$\tilde{R}_k^-=-\frac{\lambda_k}{h_k}\int_{z_{k-1}}^{z_k}(z-z_{k-1})\ddot{u}_k(z,y)dz,\ k=\overline{1,N},$$

$$\tilde{R}_k^+=-\frac{\lambda_{k+1}}{h_{k+1}}\int_{z_k}^{z_{k+1}}(z_{k+1}-z)\ddot{u}_{k+1}(z,y)dz,\ k=\overline{0,N-1},$$

$$\ddot{u}_k(z,y)=\frac{\partial^2 u_k(z,y)}{\partial y},\quad h_k=z_k-z_{k-1},\quad k=\overline{1,N},$$

$$I_k^-=-\frac{1}{h_k}\int_{z_{k-1}}^{z_k}(z-z_{k-1})q_k(z,y)dz,\ k=\overline{1,N},$$

$$I_k^+=-\frac{1}{h_{k+1}}\int_{z_k}^{z_{k+1}}(z_{k+1}-z)q_{k+1}(z,y)dz,\ k=\overline{0,N-1}.$$

We have the the exact 1-D difference scheme for given functions R_k^-,R_k^+ from (2.1).

Bem and the finite-difference scheme

The finite difference scheme (2.1) can be obtained by using the BEM. Using this method for equation (1.2) in the segment $[z_{k-1},z_k]$, we multiply the equation with the function $w(z,\xi)=|z-\xi|,\xi\in[z_{k-1},z_k]$ and partially integrate two times:

$$\lambda_k\int_{z_{k-1}}^{z_k}u_k w'' dz=\int_{z_{k-1}}^{z_k}F_k w dz+\lambda_k P_k, \tag{3.1}$$

where $P_k=(u_k w'-u_k' w)\Big|_{z_{k-1}}^{z_k}$, $w'=\dfrac{\partial w}{\partial z}$, …..

Due to $w'=sign(z-\xi)$, $w''=\delta(z-\xi)$ (the Dirac-delta function) we obtain the third Green formula for the 1-D case:

$$\lambda_k u_k(\xi,y)=\int_{z_{k-1}}^{z_k}|z-\xi|F_k w dz+\lambda_k P_k, \tag{3.2}$$

where

$$P_k=v_k(z_k)sign(z_k-\xi)-v_k'(z_k)|z_k-\xi|-v_k(z_k-1)sign(z_{k-1}-\xi)+$$
$$v_k'(z_{k-1})|z_{k-1}-\xi|,\quad v_k(z_k)\equiv u_k(z,y).$$

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For the given values $v_k(z_k), v_k(z_{k-1}), v_k'(z_k), v_k'(z_{k-1}), F_k$ it is possible from (3.2) to find $v_k(\xi) \equiv u_k(\xi, y)$ for all $\xi \in [z_{k-1}, z_k]$.

Let us consider two limit cases, when $\xi \to z_{k-1}$ and $\xi \to z_k$. Then we have 2 equations for BEM in the following form:

$$\begin{cases} \lambda_k v_k(z_{k-1}) = \lambda_k \left(v_k(z_k) - h_k v_k'(z_k)\right) + h_k R_k^- \\ \lambda_k v_k(z_k) = \lambda_k \left(v_k(z_{k-1}) + h_k v_k'(z_{k-1})\right) + h_k R_k^+. \end{cases} \qquad (3.3)$$

After substituting k by $k+1$ in second equation (3.3), then dividing this expressions respectively by h_k, h_{k+1} and applying the conditions of continuity $v_k(z_k) = v_{k+1}(z_k), \lambda_k v_k'(z_k) = \lambda_{k+1} v_{k+1}'(z_k)$, $k = \overline{1, N-1}$ we obtain the difference equations (2.1) for $k = \overline{1, N-1}$, $\left(v_k(z_k) = u_k, v_k(z_{k-1}) = u_{k-1}, v_{k+1}(z_{k+1}) = u_{k+1}\right)$. The first equation of (2.1) is obtained from the second equation (3.3), if $k = 1, \left(v_1(z_1) = u_1, v_1(z_0) = u_0\right)$, but the last equation - from first equation (3.3), if $k = N, \left(v_N(z_{N-1}) = u_{N-1}, v_N(z_N) = u_N\right)$, (the boundary conditions (1.3) must be used).

We can obtain the values $u_{k-1}, u_k, k = \overline{1, N}$ from the finite difference scheme (2.1), but from (3.3) – the values $v_k'(z_{k-1}), v_k'(z_k)$ in the grid points

$$v_k'(z_{k-1}) = \left(v_k(z_k) - v_k(z_{k-1})\right)\lambda_k^{-1} - \lambda_k^{-1} R_{k-1}^+, \; k = \overline{1, N},$$

$$v_k'(z_k) = \left(v_k(z_k) - v_k(z_{k-1})\right)\lambda_k^{-1} + \lambda_k^{-1} R_k^-, \; k = \overline{1, N}. \qquad (3.4)$$

Then in the interior points of Ω we have

$$u_k(z, y) = P_3(z, y) + \partial^4 u_k(\xi, y) / \partial z^4 \omega^2(z), \; k = \overline{1, N},$$

where $z, \xi \in [z_{k-1}, z_k]$, $\omega^2(z) = \left(z - z_{k-1}\right)^2 \left(z - z_k\right)^2 = O(h_k^4)$,

$$P_k(z, y) = u_{k-1}(y) l_{11}(z) + u_k'(z_{k-1}, y) l_{12}(z) + u_k(y) l_{21}(z) + u_k'(z_k, y) l_{22}(z), \qquad (3.5)$$

$$l_{11}(z) = \left(z - z_k\right)^2 \left(2z + z_k - 3z_{k-1}\right) / h_k^3, \quad l_{12}(z) = \left(z - z_k\right)^2 \left(z - z_{k-1}\right) / h_k^2,$$

$$l_{21}(z) = \left(z - z_{k-1}\right)^2 \left(3z_k - z_{k-1} - 2z\right) / h_k^3, \; l_{22}(z) = \left(z - z_{k-1}\right)^2 \left(z - z_k\right) / h_k^2.$$

This is the Hermite interpolation polynomial in the z -direction for solving the temperature in other interior points of Ω_k $u_k(z, y) \approx P_k(z, y)$.

For solving the loss of heat on the walls, we can calculate the heat flux functions

$$W_0 = \lambda_1 \int_0^L \frac{\partial u(0, y)}{\partial z} dy, \quad W_N = \lambda_N \int_0^L \frac{\partial u(z_N, y)}{\partial z} dy \qquad (3.6)$$

on the boundaries $z = 0$ and $z = z_N$,

where $\partial u(0, y) / \partial z \equiv u_1'(0, y) \partial u$, $\partial u(z_N, y) / \partial z \equiv u_N'(z_N, y) \partial u$.

Approximation of integrals

The value of integrals $\tilde{R}_k^+$, $\tilde{R}_k^-$ is possible to find approximately with the help of kvadrature formulas of a different sort in a 2-D case and after rejection of residual members we find the system of $N + 1$ ODEs of the second order [1-3]:

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$$\begin{cases} -\lambda_1 h_1\left[(1/3)\ddot{u}_0(y)+(1/6)\ddot{u}_1(y)\right]=(\lambda_1/h_1)\left(u_1(y)-u_0(y)\right)-\alpha_1\left(u_0(y)-\theta_0\right)-I_0^+ \\ -\lambda_k h_k\left[(1/6)\ddot{u}_{k-1}(y)+(1/3)\ddot{u}_k(t)\right]-\lambda_{k+1}h_{k+1}\left[(1/3)\ddot{u}_k(y)+(1/6)\ddot{u}_{k+1}(y)\right]= \\ =(\lambda_{k+1}/h_{k+1})\left(u_{k+1}(y)-u_k(y)\right)-(\lambda_k/h_k)\left(u_k(y)-u_{k-1}(y)\right)-I_k^- - I_k^+, \quad k=\overline{1,N-1}, \\ -\lambda_N h_N\left[(1/6)\ddot{u}_{N-1}(y)+(1/3)\ddot{u}_N(y)\right]=\lambda_N/h_N\left(u_N(y)-u_{N-1}(y)\right)-\alpha_N\left(u_N(y)-\theta_N\right)-I_N^-, \end{cases} \quad (4.1)$$

where $\ddot{u}_{k-1}(y)=\ddot{u}_k(z_{k-1},y), \ddot{u}_k(y)=\ddot{u}_k(z_k,y)$.

Here the continuity conditions of functions $u_k(z,y)$, $\ddot{u}_k(z,y)$ on a joint of layers are used.

Distribution of boundary temperature at $y=L$ we find under the linear law $u^{(0)}(z)=Bz+C$, coordinating it with the boundary conditions (1.3). Thus we find the system of two algebraic equations for the determination of constants B,C in the form

$B=\left(\alpha_1\alpha_N\left(\theta_N-\theta_0\right)\right)/\left(\alpha_1\left(\lambda_N+\alpha_N z_N\right)+\alpha_N\lambda_1\right), C=\theta_0+\left(\lambda_1/\alpha_1\right)B$.

From Dirichlet boundary conditions we obtain $B=\left(\theta_N-\theta_0\right)/z_N, C=\theta_0$. Then the boundary conditions of the system (4.1) can be presented in the form $\dot{u}_k(0)=0, u_k(L)=u^{(0)}(z_k), k=\overline{0,N}$.

Analytical solution and numerical results

Let`s consider numerical experiments, with a case $N=3, q_1=q_3=0, q_2=500\,W/m^3$, $\alpha_1=\alpha_3=\infty$, $u_0(y)=\theta_0$, $u_3(y)=\theta_3$ at the following values of parameters (in the wall of a house consisting of three layers: brick, metal, brick): $h_1=h_3=0.4\,m$, $h_2=0.2\,m$ - the thickness of layers, $l=1.5\,m$ - the half of the width of the metal – layer, $L=2\,m$ - the half of the width of the wall, $\lambda_1=\lambda_3=1.0\,W/m\cdot K$, $\lambda_2=50\,W/m\cdot K$, $\theta_0=290\,K$, $\theta_3=250\,K$ - the air temperatures in Kelvin degree (Fig. 1.).

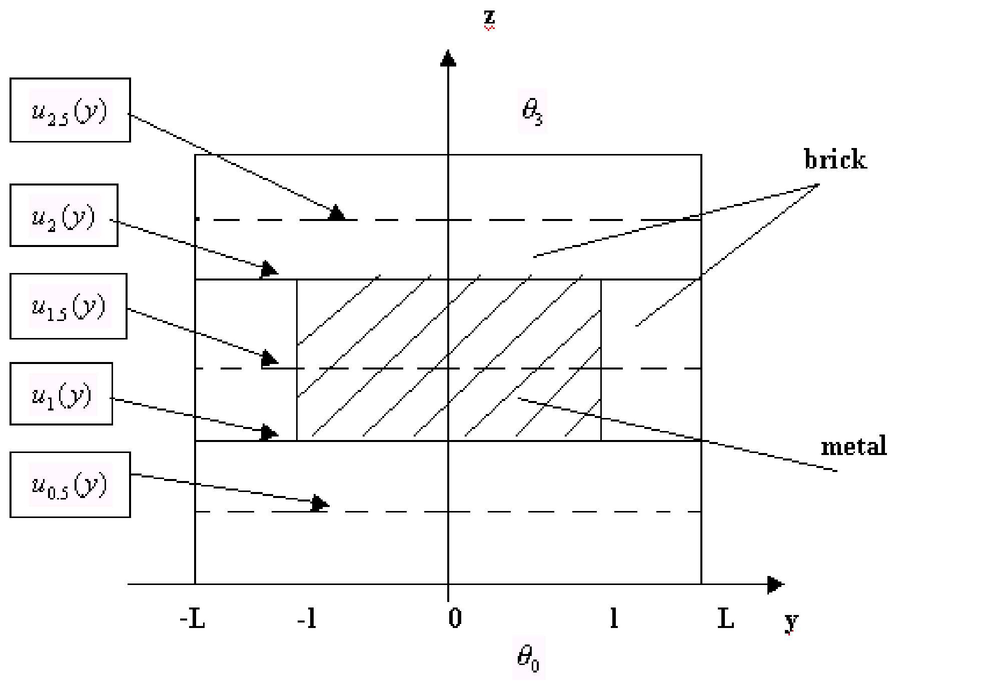

Fig. 1. The wall of a house consisting of three layers: brick, metal, brick

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In this case we have the system (4.1) of two ODEs in following form:

$$\begin{cases} -\frac{1}{3}\lambda_1 h_1 \ddot{u}_1(y) - \frac{1}{6}\lambda_2 h_2\left(-2\ddot{u}_1(y)+\ddot{u}_2(y)\right) = \frac{\lambda_2}{h_2}\left(u_2(y)-u_1(y)\right) - \frac{\lambda_1}{h_1}\left(u_1(y)-\theta_0\right) - I_1^+ \\ -\frac{1}{6}\lambda_2 h_2\left(\ddot{u}_1(y)+2\ddot{u}_2(y)\right) - \frac{1}{3}\lambda_3 h_3 \ddot{u}_2(y) = \\ = \frac{\lambda_3}{h_3}\left(\theta_3 - u_2(y)\right) - \frac{\lambda_2}{h_2}\left(u_2(y)-u_1(y)\right) - I_2^-, \end{cases}$$

(5.1)

where $I_1^+ = I_1^- = -\frac{1}{2} l_2 h_2$, $u_1'(0) - u_2'(0) = 0,\ y \in (0, L)$, $u_1(L) = \left(\theta_3 h_1 + \theta_0\left(h_1 + h_2\right)\right)\left(2h_1 + h_2\right)^{-1}$, $u_2(L) = \left(\theta_0 h_1 + \theta_3\left(h_1 + h_2\right)\right)\left(2h_1 + h_2\right)^{-1}$ or in the 1-D case $\dot{u}_1(L) = \dot{u}_2(L) = 0$.

It is possible to obtain the solution of boundary – value problem (5.1) (if $q_2 = q_2(y) \neq 0$ $y \in [0, l]\ l < L$) in the form

$$u_1(y) = C_2 ch(\mu_1 y) + C_4 ch(\mu_3 y) + d_1,\quad u_2(y) = C_2 ch(\mu_1 y) - C_4 ch(\mu_3 y) + d_2, \qquad (5.2)$$

where $\mu_1 = \left(6\lambda_1 / \left(h_1\left(2\lambda_1 h_1 + 3\lambda_2 h_2\right)\right)\right)^{\frac{1}{2}}$, $\mu_3 = \left(\left(6\left(2\lambda_2 h_1 + \lambda_1 h_2\right)\right) / \left(h_1 h_2\left(2\lambda_1 h_2 + \lambda_2 h_2\right)\right)\right)^{\frac{1}{2}}$,
$a_2 = \lambda_1 h_1^{-1} + \lambda_2 h_2^{-1}$, $b_2 = -\lambda_2 h_2^{-1}$, $c_1 = \lambda_1 h_1^{-1}\theta_0 + 0.5 q_2 h_2$, $c_2 = \lambda_1 h_1^{-1}\theta_3 + 0.5 q_2 h_2$,
$c_0 = a_2^2 - b_2^2 = \lambda_1^2 h_1 + 2\lambda_1\lambda_2\left(h_1 h_2\right)^{-1}$,
$d_1 = \left(a_2 c_1 - b_2 c_2\right) c_0^{-1}$, $d_2 = \left(a_2 c_2 - b_2 c_1\right) c_0^{-1}$, $C_2 = -\left(d_1 + d_2 - u_1(L) - u_2(L)\right) / \left(2ch(\mu_1 L)\right)$,
$C_4 = -\left(d_1 - d_2 - u_1(L) + u_2(L)\right) / \left(2ch(\mu_3 L)\right)$ (5.3)

If $\dot{u}_1(L) = \dot{u}_2(L) = 0$ (the 1-D case), then $C_2 = C_4 = 0$ and $u_1(y) = d_1 = \left(p_2\theta_0 + p_1\theta_3\right) / p_3 + Q$,
$u_2(y) = d_2 = \left(p_1\theta_0 + p_2\theta_3\right) / p_3 + Q$, where $p_1 = \lambda_2 h_1 = 20$, $p_2 = \lambda_1 h_2 + \lambda_2 h_1 = 20.2$, $p_3 = p_1 + p_2 = 40.2$,
$Q = \frac{1}{2} q_2 h_2 h_1 / \lambda_1 = 20.$ Therefore, $u_1 = 270.1 + Q = 290.1$, $u_2 = 268.8 + Q = 288.8$.

It is possible to obtain the solution of boundary – value problem (5.1) (if $q_2 = 0$, $\lambda_2 = \lambda_1$, $y \in [l, L]$) in the form

$$\begin{cases} u_1^\bullet(y) = C_1^\bullet sh(\mu_1^\bullet y) + C_2^\bullet ch(\mu_1^\bullet y) + C_3^\bullet sh(\mu_3^\bullet y) + C_4^\bullet ch(\mu_3^\bullet y) + d_1^\bullet \\ u_2^\bullet(y) = C_1^\bullet sh(\mu_1^\bullet y) + C_2^\bullet ch(\mu_1^\bullet y) - C_3^\bullet sh(\mu_3^\bullet y) - C_4^\bullet ch(\mu_3^\bullet y) + d_2^\bullet \end{cases} \qquad (5.4)$$

where, $\mu_1^\bullet = \left(6 / \left(h_1\left(2h_1 + 3h_2\right)\right)\right)^{\frac{1}{2}}$, $\mu_3^\bullet = \left(6 / \left(h_1\lambda_2\right)\right)^{\frac{1}{2}}$, $a_2^\bullet = \lambda_1\left(h_1^{-1} + h_2^{-1}\right)$, $b_2^\bullet = -\lambda_1 h_2^{-1}$,
$c_0^\bullet = \lambda_1^2\left(h_1^{-2} + 2\left(h_1 h_2\right)^{-1}\right)$, $c_1^\bullet = \lambda_1 h_1^{-1}\theta_0$, $c_2^\bullet = \lambda_1 h_1^{-1}\theta_3$, $d_1^\bullet = \left(a_2^\bullet c_1^\bullet - b_2^\bullet c_2^\bullet\right)\left(c_0^\bullet\right)^{-1}$,
$d_2^\bullet = \left(a_2^\bullet c_2^\bullet - b_2^\bullet c_1^\bullet\right)\left(c_0^\bullet\right)^{-1}$, $\phi_k = \frac{\lambda^\bullet \mu_k}{\lambda_1 \mu_k^\bullet}$, $k = 1,3,$ $d_1^+ = (1/2)\left(d_1 - d_1^\bullet + d_2 - d_2^\bullet\right)$,
$d_1^- = (1/2)\left(d_1 - d_1^\bullet - d_2 + d_2^\bullet\right)$,
$A_1 = ch(\mu_1 l) ch\left((L - l)\mu_1^\bullet\right) + \phi_1 sh(\mu_1 l) sh\left((L - l)\mu_1^\bullet\right)$,
$A_2 = ch(\mu_3 l) ch\left((L - l)\mu_3^\bullet\right) + \phi_3 sh(\mu_3 l) sh\left((L - l)\mu_3^\bullet\right)$,

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$$C_2 = (1/2A_1)\left(u_1(L) + u_2(L) - 2d_1^+ ch\left((L-l)\mu_1^\bullet\right) - d_1^\bullet - d_2^\bullet\right),$$
$$C_4 = (1/2A_1)\left(u_1(L) - u_2(L) - 2d_1^- ch\left((L-l)\mu_3^\bullet\right) - d_1^\bullet + d_2^\bullet\right), \quad (5.5)$$
$$C_1^\bullet = -d_1^+ sh(\mu_1^\bullet l) + C_2\left(\phi_1 sh(\mu_1 l)ch(\mu_1^\bullet l) - ch(\mu_1 l)sh(\mu_1^\bullet l)\right),$$
$$C_2^\bullet = d_1^+ ch(\mu_1^\bullet l) + C_2\left(\phi_1 ch(\mu_1 l)ch(\mu_1^\bullet l) - \phi_1 sh(\mu_1^\bullet l)sh(\mu_1^\bullet l)\right),$$
$$C_3^\bullet = -d_1^- sh(\mu_3^\bullet l) + C_4\left(\phi_3 sh(\mu_3 l)ch(\mu_3^\bullet l) - ch(\mu_3 l)sh(\mu_3^\bullet l)\right),$$
$$C_4^\bullet = d_1^- ch(\mu_3^\bullet l) + C_4\left(ch(\mu_3 l)ch(\mu_3^\bullet l) - \phi_3 ch(\mu_3^\bullet l)sh(\mu_3 l)\right).$$

In calculations $d_1^\bullet$, $d_2^\bullet$ under formulas (5.3), at $y \in (l, L)$ $(l < L)$ loss of significant figures was observed, for example, at $q_2 = 500$, $y = 1.8$ we find $u_1(y) = 278.256305$ (at calculation using up to 20 significant figures), $u_1(y) = 278.2$ (at calculation using up to 15 significant figures), $u_1(y) = 304$ (at calculation using up to 12 significant figures - it is visible, that all received figures are incorrect). Therefore it is more expedient to use formulas

$$u_1^\bullet(y) = d_1^+ ch\left((y-l)\mu_1^\bullet\right) + d_1^- ch\left((y-l)\mu_3^\bullet\right) + C_2 A_1(y) + C_4 A_2(y) + d_1^\bullet,$$
$$u_1^\bullet(y) = d_1^+ ch\left((y-l)\mu_1^\bullet\right) - d_1^- ch\left((y-l)\mu_3^\bullet\right) + C_2 A_1(y) - C_4 A_2(y) + d_2^\bullet,$$

where $A_1(y) = ch(\mu_1 l)ch\left((y-l)\mu_1^\bullet\right) + \lambda_{av}^{(1)} sh(\mu_1 l)sh\left((y-l)\mu_1^\bullet\right)$,

$A_2(y) = ch(\mu_3 l)ch\left((y-l)\mu_3^\bullet\right) + \lambda_{av}^{(2)} sh(\mu_3 l)sh\left((y-l)\mu_3^\bullet\right)$, instead of (5.4).

If $l = L$, $\mu_1^\bullet = \mu_1$, $\mu_3^\bullet = \mu_3$, then the constants C_2, C_4 from (5.5) are equal C_2, C_4 from (5.3) and we can use only formulas(5.2). In the interior points we can used the interpolatioin with cubic polinomials (3.5), where

$$u_1'(0,y) = \left(u_1(y) - \theta_0\right)h_1^{-1} + (h_1/6)B_1(y),\ u_1'(z_1,y) = \left(u_1(y) - \theta_0\right)h_1^{-1} - (h_1/3)B_1(y),$$
$$u_2'(z_1,y) = \left(u_2(y) - u_1(y)\right)h_2^{-1} + (h_2/6)\left(2B_1(y) + B_2(y)\right) + (q_2 h_2)/(2h_2),$$
$$u_2'(z_2,y) = \left(u_2(y) - u_1(y)\right)h_2^{-1} - (h_2/6)\left(B_1(y) + 2B_2(y)\right) - (q_2 h_2)/(2h_2),$$
$$u_3'(z_2,y) = \left(\theta_3 - u_2(y)\right)h_3^{-1} + (h_3/3)B_2(y),\ u_3'(z_3,y) = \left(\theta_3 - u_2(y)\right)h_3^{-1} - (h_3/6)B_2(y), \quad (5.6)$$
$$B_1(y) = C_2\mu_1^2 ch(\mu_1 y) + C_4\mu_3^2 ch(\mu_3 y),\ B_2(y) = C_2\mu_1^2 ch(\mu_1 y) - C_4\mu_3^2 ch(\mu_3 y).$$

If $l < L$, then for $y > L$ in the formulas (5.6) the functions B_1, B_2 can be replaced with $B_1^\bullet, B_2^\bullet$, and u_1, u_2 with $u_1^\bullet, u_2^\bullet$, where

$$B_1^\bullet = d_1^+(\mu_1^\bullet)^2 ch\left((y-l)\mu_1^\bullet\right) + d_1^-(\mu_3^\bullet)^2 ch\left((y-l)\mu_3^\bullet\right) + C_2(\mu_1^\bullet)^2 A_1(y) + C_4(\mu_3^\bullet)^2 A_2(y),$$
$$B_2^\bullet = d_1^+(\mu_1^\bullet)^2 ch\left((y-l)\mu_1^\bullet\right) - d_1^-(\mu_3^\bullet)^2 ch\left((y-l)\mu_3^\bullet\right) + C_2(\mu_1^\bullet)^2 A_1(y) - C_4(\mu_3^\bullet)^2 A_2(y).$$

For the flux functions on the borders (3.6) we obtain

$$W_0 = \lambda_1\left(W_0^{(1)} + W_0^{(2)}\right),\ W_3 = \lambda_3\left(W_3^{(1)} + W_3^{(2)}\right), \quad (5.7)$$

where

$$W_0^{(1)} = \int_0^l u_1'(0,y)dy = lh_1^{-1}\left(d_1 - \theta_0\right) + C_2K_1 + C_4K_3,$$
$$W_3^{(1)} = \int_0^l u_1'(0,y)dy = lh_3^{-1}\left(\theta_3 - d_2\right) - C_2K_1 + C_4K_3,$$
$$W_0^{(2)} = \int_0^l u_1'(0,y)dy = (L-l)h_1^{-1}\left(d_1^\bullet - \theta_0\right) + d_1^+K_1^\bullet + d_1^-K_3^\bullet + C_2D_1 + C_4D_3,$$
$$W_3^{(2)} = \int_0^l u_1'(0,y)dy = (L-l)h_3^{-1}\left(\theta_3 - d_2^\bullet\right) - d_1^+K_1^\bullet + d_1^-K_3^\bullet - C_2D_1 + C_4D_3,$$

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where $K_1 = sh(\mu_1 l)\left[(h_1\mu_1)^{-1} + h_1\mu_1/6\right]$, $K_3 = sh(\mu_3 l)\left[(h_1\mu_3)^{-1} + h_1\mu_3/6\right]$,
$K_1^\bullet = sh\left((L-l)\mu_1^\bullet\right)\left[(h_1\mu_1^\bullet)^{-1} + h_1\mu_1^\bullet/6\right]$, $K_3^\bullet = sh\left((L-l)\mu_3^\bullet\right)\left[(h_1\mu_3^\bullet)^{-1} + h_1\mu_3^\bullet/6\right]$,
$D_1 = ch(\mu_1 l)K_1^\bullet + \lambda_{av}^{(1)} sh(\mu_1 l)\left(ch\left((L-l)\mu_1^\circ\right)-1\right)K_1^0$,
$D_3 = ch(\mu_3 l)K_3^\bullet + \lambda_{av}^{(2)} sh(\mu_3 l)\left(ch\left((L-l)\mu_3^\circ\right)-1\right)K_3^0$,
$K_1^0 = \left(h_1\mu_1^\bullet\right)^{-1} + h_1\mu_1^\bullet/6$, $K_3^0 = \left(h_1\mu_3^\bullet\right)^{-1} + h_1\mu_3^\bullet/6$.

If $l = L$, $\dot{u}_1(L) = \dot{u}_2(L) = 0$ (1-D case), then $C_2 = C_4 = 0$, $W_0^{(2)} = W_3^{(2)} = 0$ and
$u_1'(0) = u_1'(z_1) = (\theta_3 - \theta_0)\lambda_2/p_3 + Q_0/\lambda_1 = -49.75 + Q_0$,
$u_2'(z_1) = (\theta_3 - \theta_0)\lambda_1/p_3 + Q_0/\lambda_2 = -1 + 0.02\,Q_0$,
$u_2'(z_2) = (\theta_3 - \theta_0)\lambda_1/p_3 - Q_0/\lambda_2 = -1 - 0.02\,Q_0$,
$u_3'(z_2) = u_3'(z_3) = (\theta_3 - \theta_0)\lambda_2/p_3 - Q_0/\lambda_1 = -49.75 - 0.02\,Q_0$,
where $p_3 = 2h_1\lambda_2 + h_2\lambda_1 = 40.2$, $Q_0 = (q_2 h_2)/2 = 50$.

Therefore $W_0 = L\left[(\theta_3 - \theta_0)\lambda_1\lambda_2/p_3 + Q_0\right] = L(-49.75 + Q_0)$, $W_3 = L(-49.75 - Q_0)$.

We see, that at heating an average layer, loss of heat inside a house are practically equal to zero. Calculations were made by means of mathematical computer-system MAPLE-5, in the Table 1, Figure 2 we can see the dependence of temperature $u_{0.5}(y) = u(h_1/2, y)$, $u_1(y) = u(h_1, y)$, $u_{1.5}(y) = u(h_1 + h_2/2, y)$, $u_2(y) = u(z_2, y)$, $u_{2.5}(y) = u(z_2 + h_3/2, y)$ $(z_2 = h_1 + h_2)$ on the coordinate $y \in (0, L)$ with. $q_2 = 500\,W/m^3$. We have, $W_0 = -23.68$,. $W_3 = -171.83$. If $l = L$ and $\dot{u}_1(L) = \dot{u}_2(L) = 0$, then $u_{0.5} = (u_1 + \theta_0)/2 \approx 280 + Q/2$, $u_1 = 270.1 + Q$, $u_{1.5} = (u_1 + u_2)/2 + 0.005\,h_2\,Q_0$, $u_2 = 268.8 + Q$, $u_{2.5} = (u_2 + \theta_3)/2 \approx 260 + Q/2$, $Q = 20$, $Q_0 = 50$.

Table 1.

Depending on temperatures with $q_2 = 500\frac{W}{m^3}$ **,** $u_0 = \theta_0 = 290^0\,K$ **,** $u_3 = \theta_3 = 250^0\,K$

y	$u_{0.5}$	u_1	$u_{1.5}$	u_2	$u_{2.5}$
0	**288.77**	**287.57**	**287.51**	**287.37**	**268.67**
0.2	**288.76**	**287.54**	**287.49**	**287.34**	**268.66**
0.4	**288.72**	**287.47**	**287.41**	**287.27**	**268.62**
0.6	**288.66**	**287.34**	**287.28**	**287.14**	**268.56**
0.8	**288.57**	**287.16**	**287.10**	**286.96**	**268.47**
1.0	**288.45**	**286.92**	**286.87**	**286.72**	**268.35**
1.2	**288.30**	**286.62**	**286.57**	**286.42**	**268.19**
1.4	**288.16**	**286.27**	**286.20**	**286.04**	**267.96**
1.6	**286.72**	**283.47**	**281.67**	**278.68**	**266.73**
1.8	**284.41**	**278.26**	**174.77**	**270.81**	**261.10**
2.0	**282.00**	**274.00**	**270.00**	**266.00**	**258.00**

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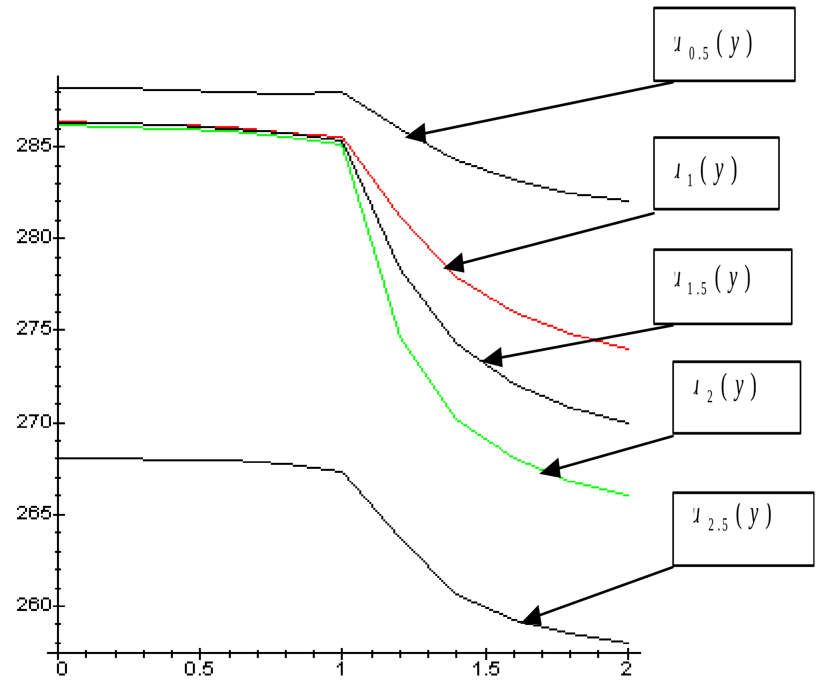

Fig. 2. Depending on temperatures with $q_2 = 500\frac{W}{m^3}$,

$u_0 = \theta_0 = 290^0 K$, $u_3 = \theta_3 = 250^0 K$, $l = 1.0\,m$

Conclusion

In the given paper the used method allows to reduce a two-dimensional heat transfer problem to the system of the ordinary differential equations (5.3), solution of both it is easier to make theoretically and practically.

The considered method allows finding distribution of temperatures depending on the second coordinate on a joint of layers.

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